Light rings and causality for nonsingular ultracompact objects sourced by nonlinear electrodynamics

Sebastian Murk

OIST OKINAWA INSTITUTE OF SCIENCE AND TECHNOLOGY
沖縄科学技術大学院大学

GR 24 & Amaldi 16
Thursday, July 17, 2025
Scottish Exhibition Centre, Glasgow, UK

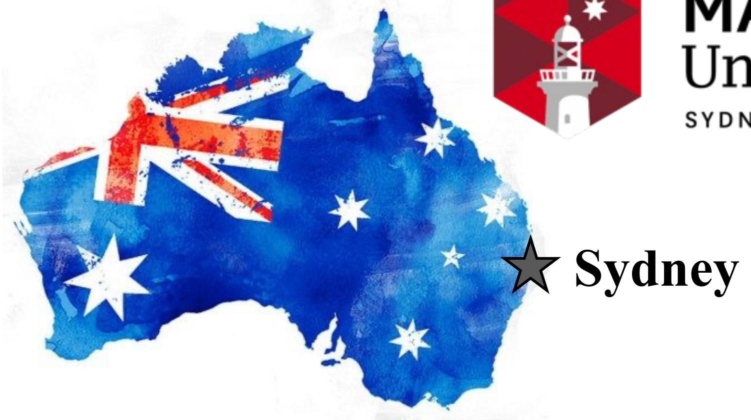
Based on Phys. Rev. D 110, 044064 (2024) / arXiv:2406.07957

PHYSICAL REVIEW D **110,** 044064 (2024)

Light rings and causality for nonsingular ultracompact objects sourced by nonlinear electrodynamics

Sebastian Murk[1,*] and Ioannis Soranidis[2,†]

[1]*Quantum Gravity Unit, Okinawa Institute of Science and Technology,*
1919-1 Tancha, Onna-son, Okinawa 904-0495, Japan
[2]*School of Mathematical and Physical Sciences, Macquarie University,*
Sydney, New South Wales 2109, Australia



MACQUARIE University
SYDNEY·AUSTRALIA

Ioannis Soranidis

Publications:

- SM, I Soranidis*
  PRD **108**, 044002 (2023)
- SM, I Soranidis*
  PRD **108**, 124007 (2023)
- SM, I Soranidis*
  PRD **110**, 044064 (2024)

Recap: Nonlinear Electrodynamics

EM field tensor: $F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu$

Invariants: $\mathcal{F} = F_{\mu\nu} F^{\mu\nu}$, $\quad \mathcal{G} = F_{\mu\nu}({}^\star F^{\mu\nu})$

$${}^\star F^{\mu\nu} := \tfrac{1}{2}\varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

NED action: $$S = \frac{1}{16\pi}\int \sqrt{-g}[R - \mathcal{L}(\mathcal{F})] d^4x$$

$$\mathcal{L}(\mathcal{F},\mathcal{G}) \equiv \mathcal{L}(\mathcal{F})$$

NED field equations: $\nabla_\mu(\mathcal{L}_\mathcal{F} F^{\mu\nu}) = 0$, $\quad \nabla_\mu({}^\star F^{\mu\nu}) = 0$

$\mathcal{L}_\mathcal{F} := \partial\mathcal{L}/\partial\mathcal{F}$ (Bianchi identities)

Coupled Einstein equations: $$G_{\mu\nu} := R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$T_{\mu\nu} = \frac{1}{4\pi}\left(\mathcal{L}_\mathcal{F} F_{\mu\rho} F^{\rho}{}_{\nu} - \frac{1}{4} g_{\mu\nu}\mathcal{L}\right)$$

Maxwell weak-field limit: $\lim\limits_{\mathcal{F}\to 0}\mathcal{L} \simeq \mathcal{F}$, $\quad \lim\limits_{\mathcal{F}\to 0}\mathcal{L}_\mathcal{F} \simeq 1$

Brief review article:

Sorokin
Fortschr. Phys. **70**, 2200092 (2022)

NED in spherical symmetry & Birefringence

Spherically symmetric metric: $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2$

$$d\Omega^2 \equiv d\theta^2 + \sin^2\theta d\phi^2$$

$\Rightarrow$ Nonvanishing components: $F_{tr} = -F_{rt}$ (radial E field) $Q_e = -r^2 \mathcal{L}_\mathcal{F} F_{tr}$

$F_{\theta\phi} = -F_{\phi\theta}$ (radial B field) $Q_m = -\dfrac{F_{\theta\phi}}{\sin\theta}$

Symmetries of EMT: ${T_t}^t = {T_r}^r$ (invariance under boosts in radial direction)

${T_\theta}^\theta = {T_\phi}^\phi$ (spherical symmetry)

Birefringence: The two photon modes propagate w.r.t. distinct metrics. In our case $\mathcal{L}(\mathcal{F},\mathcal{G}) \equiv \mathcal{L}(\mathcal{F})$,

background metric tensor: $g^{\mu\nu}$

effective metric tensor: $\bar{g}^{\mu\nu} = g^{\mu\nu} - 4\dfrac{\mathcal{L}_{\mathcal{F}\mathcal{F}}}{\mathcal{L}_\mathcal{F}} {F^\mu}_\rho F^{\rho\nu}$

Magnetic solutions in NED

Solutions with electric charge: $Q_e \neq 0$ ➡ $\mathcal{L}(\mathcal{F})$ either non-Maxwellian or nonlocal

Bronnikov
Phys. Rev. D **63**, 044005 (2001)

Magnetic solutions: $Q_e = 0$ ➡ $F_{tr} = 0$

$Q_m \neq 0$

dimensionless constants

integration const.

➡ Lagrangian density: $\mathcal{L}(\mathcal{F}) = \dfrac{4\mu(\alpha\mathcal{F})^{\frac{\nu+3}{4}}}{\alpha[1+(\alpha\mathcal{F})^{\frac{\nu}{4}}]^{1+\frac{\mu}{\nu}}}$ where $Q_m = \dfrac{q^2}{\sqrt{2\alpha}}, \quad \mathcal{F} = \dfrac{2Q_m^2}{r^4}$

Fan and Wang
Phys. Rev. D **94**, 124027 (2016)

$\alpha > 0$ related to grav. mass $M \equiv q^3/\alpha$

(coincides with em.-induced mass)

Popular models:

$(\mu,\nu) = (3,1)$	Cadoni *et al.*	Cadoni *et al.* Phys. Rev. D **107**, 044038 (2023)
$(\mu,\nu) = (3,2)$	Bardeen	Bardeen GR5 Conference Proc. (Tbilisi, 1968).
$(\mu,\nu) = (3,3)$	Hayward	Hayward Phys. Rev. Lett. **96**, 031103 (2006)

Magnetic solutions: Effective metric

Solutions with electric charge: $Q_e \neq 0$ ➡ $\mathcal{L}(\mathcal{F})$ either non-Maxwellian or nonlocal

Bronnikov
Phys. Rev. D **63**, 044005 (2001)

Magnetic solutions: $Q_e = 0$ ➡ $F_{tr} = 0$

$Q_m \neq 0$

$$\bar{g}^{\mu\nu} = g^{\mu\nu} - 4\frac{\mathcal{L}_{\mathcal{F}\mathcal{F}}}{\mathcal{L}_{\mathcal{F}}} F^{\mu}{}_{\rho} F^{\rho\nu}$$

$$\bar{g}^{\mu\rho}\bar{g}_{\rho\nu} = \delta^{\mu}{}_{\nu}$$

➡ Effective metric: $\bar{g}_{tt} = g_{tt}, \qquad \bar{g}_{rr} = g_{rr},$

$$\bar{g}_{\theta\theta} = g_{\theta\theta}\left(1 + \frac{2\mathcal{F}\mathcal{L}_{\mathcal{F}\mathcal{F}}}{\mathcal{L}_{\mathcal{F}}}\right)^{-1},$$

$$\bar{g}_{\phi\phi} = g_{\phi\phi}\left(1 + \frac{2\mathcal{F}\mathcal{L}_{\mathcal{F}\mathcal{F}}}{\mathcal{L}_{\mathcal{F}}}\right)^{-1}$$

Regular black holes and nonsingular horizonless UCOs

Equivalently: $f(r) = 1 - \frac{2Mr^{\mu-1}}{(r^\nu + q^\nu)^{\mu/\nu}}$

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2$$

$$M \equiv q^3/\alpha$$

$$\alpha \propto q \propto \ell$$

Solving $f(r) = 0$ may result in the following:

(I) Two roots $[\ell < \ell_c]$, which corresponds to an RBH with an inner and an outer horizon.

(II) One root $[\ell = \ell_c]$, which corresponds to an extremal RBH with one degenerate horizon.

(III) No roots $[\ell > \ell_c]$, which corresponds to a nonsingular horizonless UCO.

critical length ℓ_c

$$r_+(\ell_c) \equiv r_-(\ell_c)$$

Limiting behaviors

$$\mathcal{L}(\mathcal{F}) = \frac{4\mu(\alpha\mathcal{F})^{\frac{\nu+3}{4}}}{\alpha[1+(\alpha\mathcal{F})^{\frac{\nu}{4}}]^{1+\frac{\mu}{\nu}}}$$

where $Q_m = \frac{q^2}{\sqrt{2\alpha}}, \quad \mathcal{F} = \frac{2Q_m^2}{r^4}$ $\qquad \alpha \propto q \propto \ell$

	$\lim\limits_{\mathcal{F}\to 0}$	$\lim\limits_{\ell\to 0}$	$\lim\limits_{r\to\infty}$	
Cadoni *et al.* $(\mu,\nu)=(3,1)$	$\sim\mathcal{O}(\mathcal{F})$	$\bar{g}_{\mu\mu}=g_{\mu\mu}$	$\bar{g}_{\mu\mu}=g_{\mu\mu}$	✅
Bardeen $(\mu,\nu)=(3,2)$	$\sim\mathcal{O}(\mathcal{F}^{5/4})$	$\bar{g}_{\mu\mu}\neq g_{\mu\mu}$	$\bar{g}_{\mu\mu}\neq g_{\mu\mu}$	❌
Hayward $(\mu,\nu)=(3,3)$	$\sim\mathcal{O}(\mathcal{F}^{3/2})$	$\bar{g}_{\mu\mu}\neq g_{\mu\mu}$	$\bar{g}_{\mu\mu}\neq g_{\mu\mu}$	❌

Light Rings

$$L_p = \frac{1}{2} g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu = \frac{u^2}{2} = \frac{1}{2}\left(g_{tt}\dot{t}^2 + g_{rr}\dot{r}^2 + g_{\phi\phi}\dot{\phi}^2\right)$$

Work in equatorial plane: $\theta = \pi/2,\ \theta = 0$

Four-velocity: $u^\mu = (\dot{t}, \dot{r}, 0, \dot{\phi})$, $u^2 := u_\mu u^\mu$

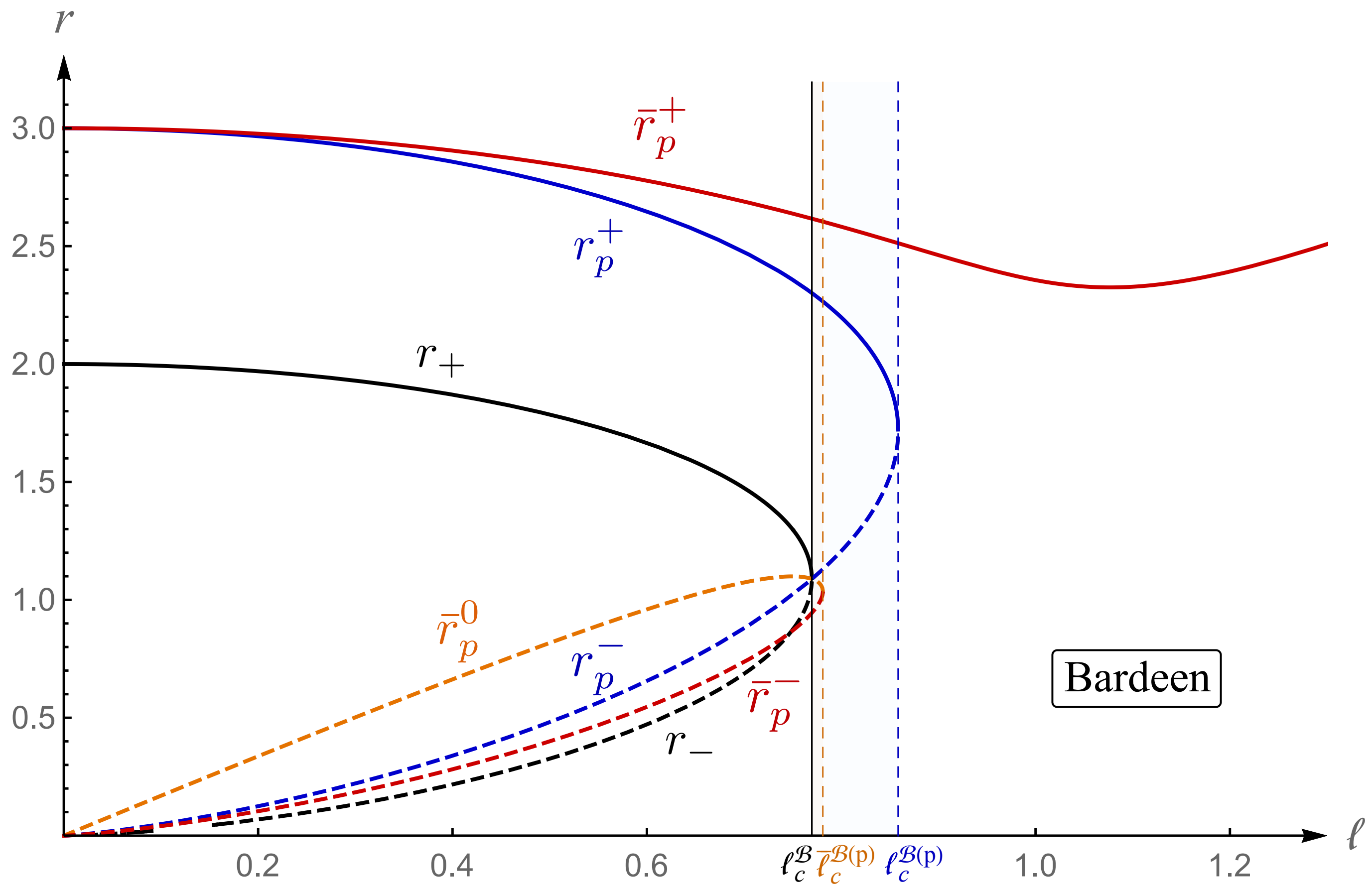
Noether's theorem: $\frac{\partial L_p}{\partial \dot{t}} = g_{tt}\dot{t} = -E$, $\frac{\partial L_p}{\partial \dot{\phi}} = g_{\phi\phi}\dot{\phi} = L$

LRs are null geodesics: $g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu = 0$ $\Rightarrow$ $\dot{r}^2 + V(r) = 0,$ $V(r) := \frac{E^2}{g_{tt}g_{rr}} + \frac{L^2}{g_{rr}g_{\phi\phi}}$

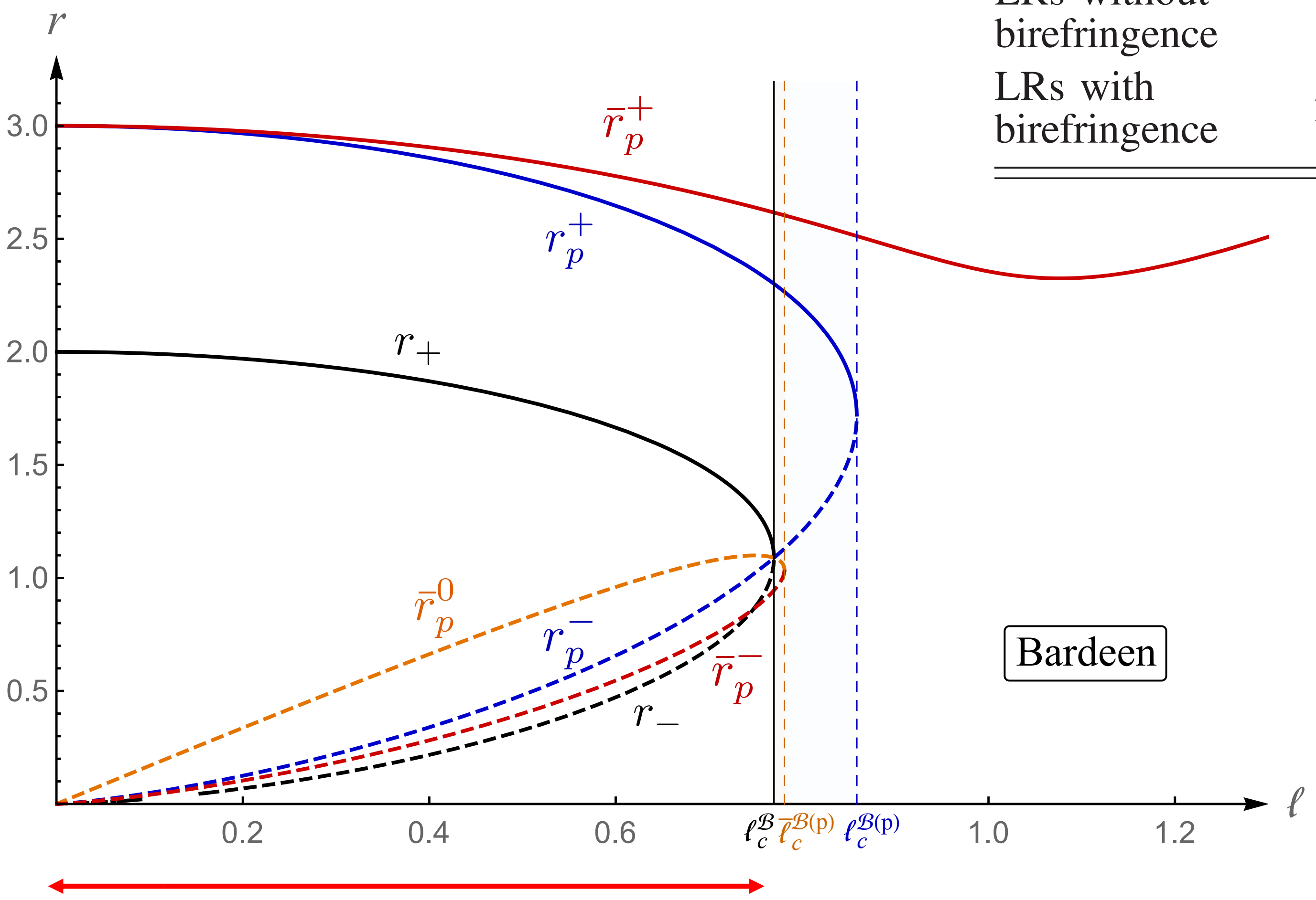
LR location is determined by $\dot{r} = 0$ and $\ddot{r} = 0$ $\Leftrightarrow$ $V(r) = 0$ and $V'(r) = 0$

and analogously for the effective metric!

Light Rings: Bardeen Model

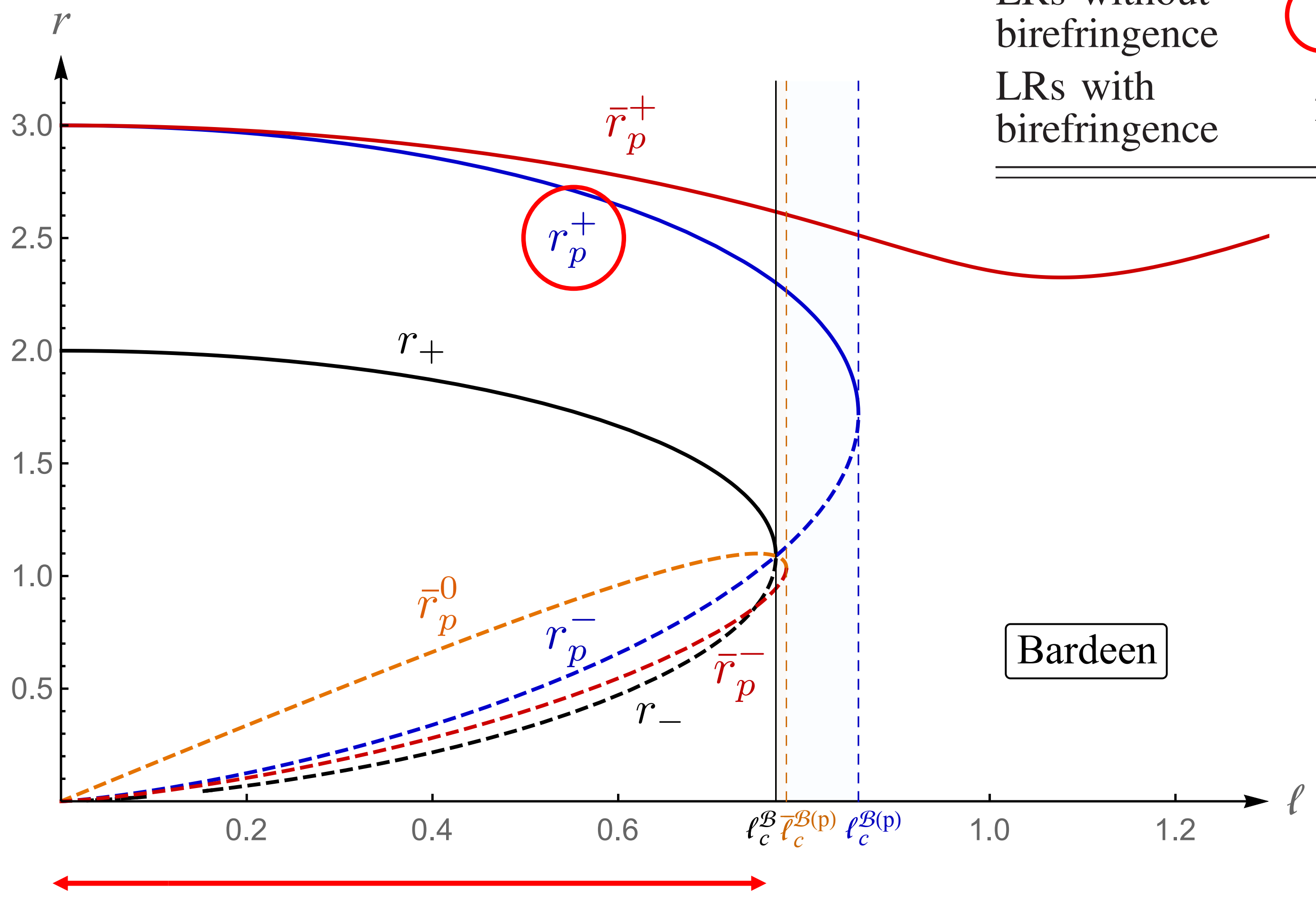


Light Rings: Bardeen Model



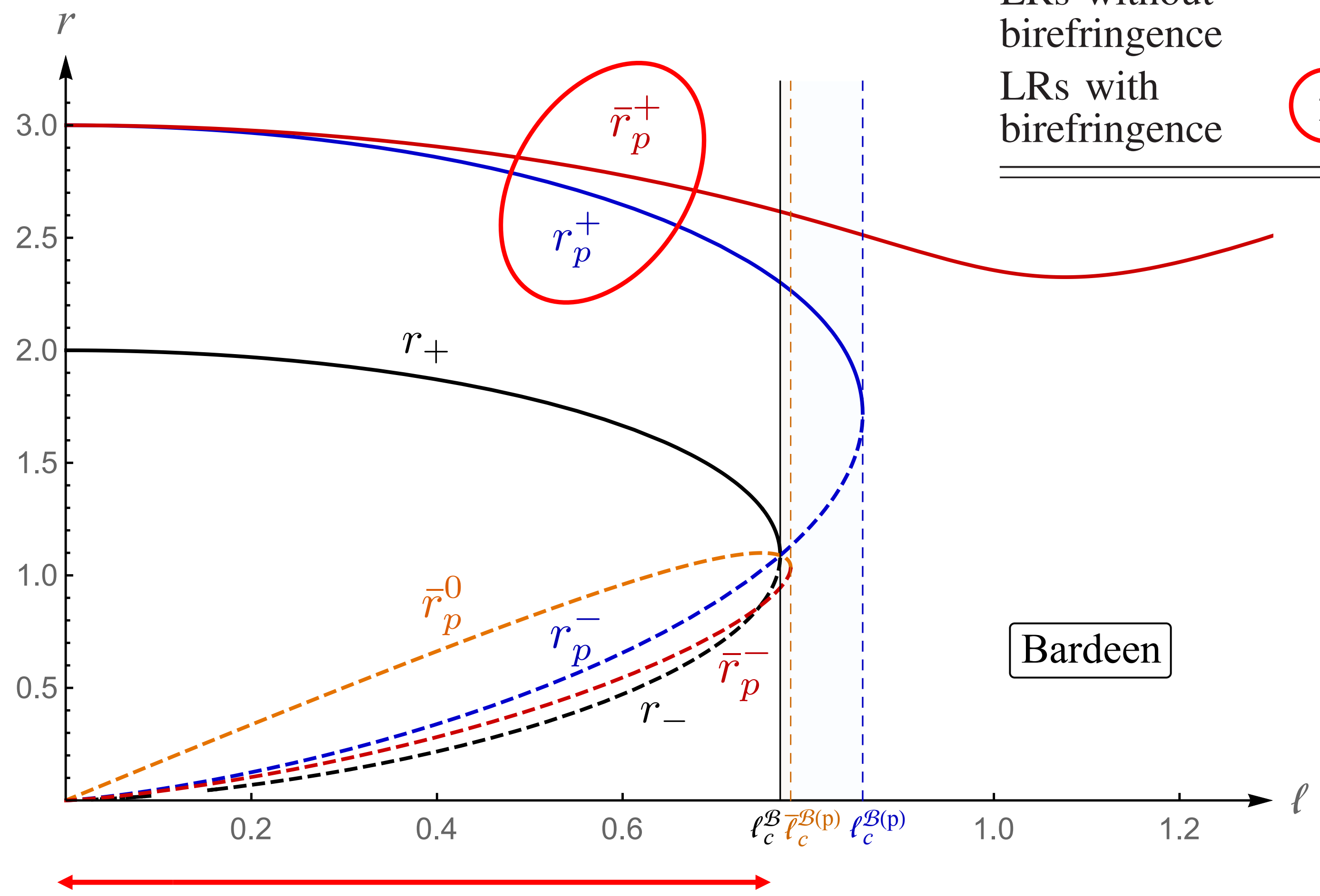
	$0 < \ell < \ell_c$
Nonsingular UCO type	RBH
LRs without birefringence	1
LRs with birefringence	2

Light Rings: Bardeen Model



	$0 < \ell < \ell_c$
Nonsingular UCO type	RBH
LRs without birefringence	1
LRs with birefringence	2

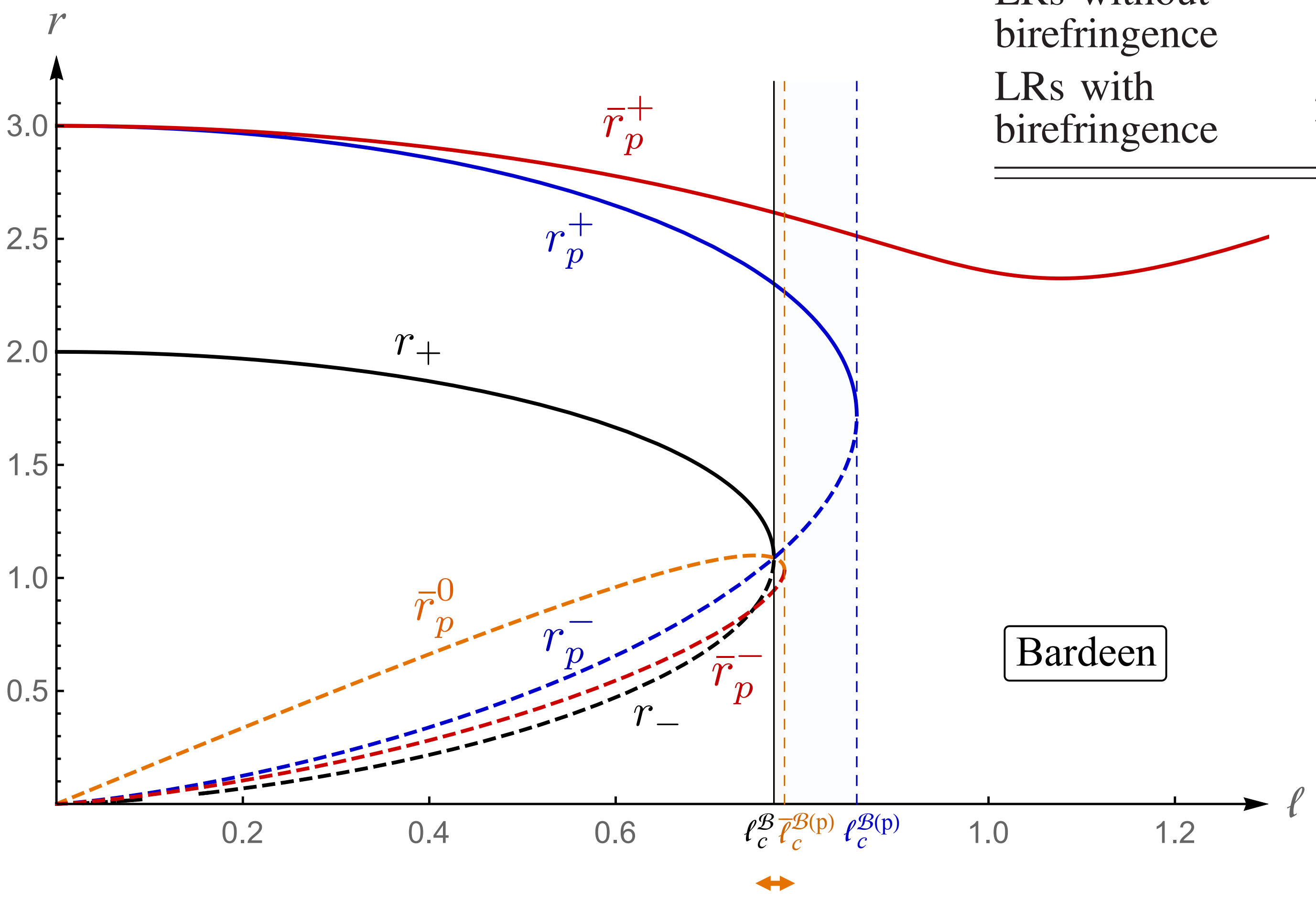
Light Rings: Bardeen Model



	$0 < \ell < \ell_c$
Nonsingular UCO type	RBH
LRs without birefringence	1
LRs with birefringence	2

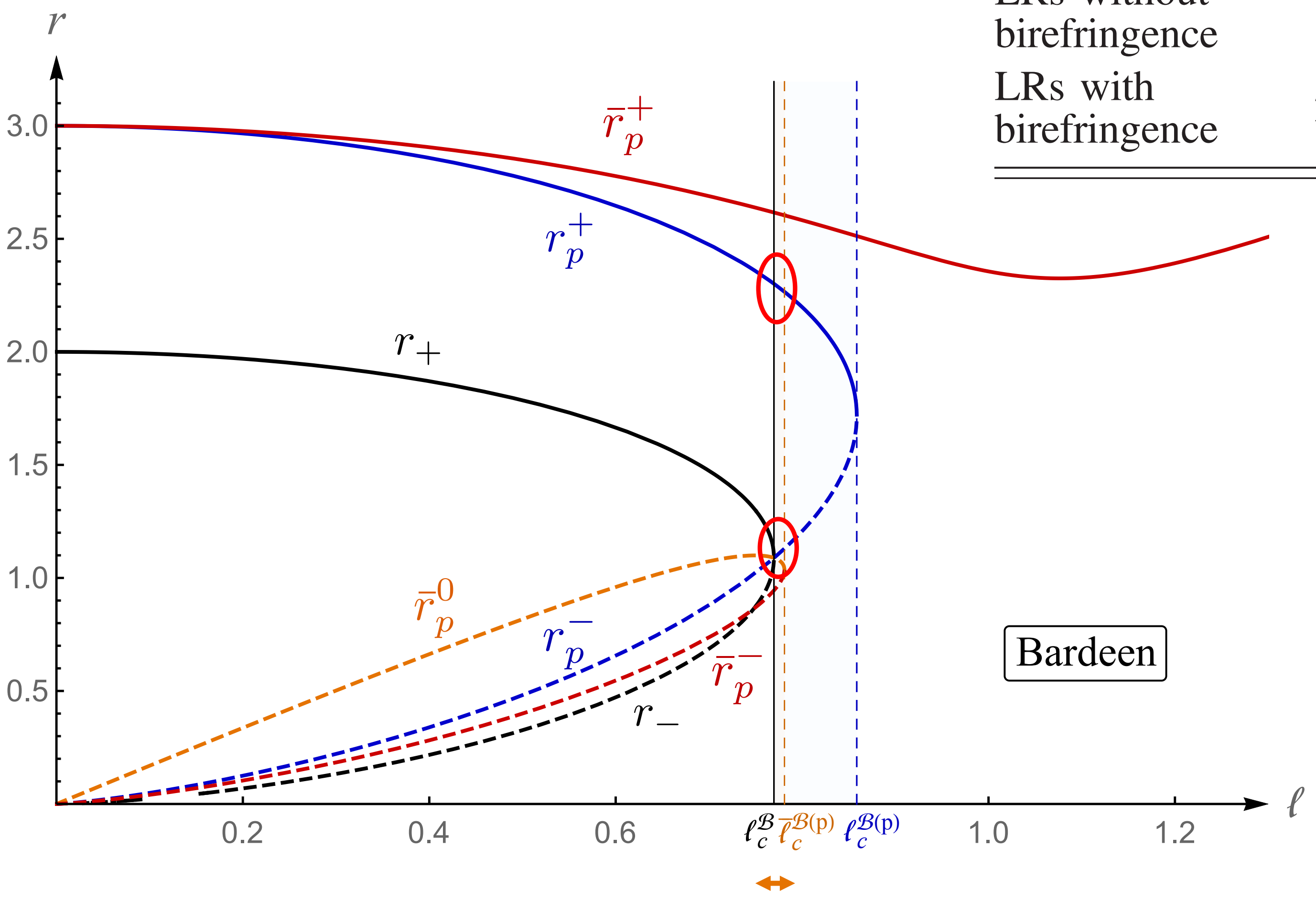
Light Rings: Bardeen Model

	$0 < \ell < \ell_c$	$\ell_c < \ell < \overline{\ell}_c^{(p)}$
Nonsingular UCO type	RBH	Horizonless
LRs without birefringence	1	2
LRs with birefringence	2	5



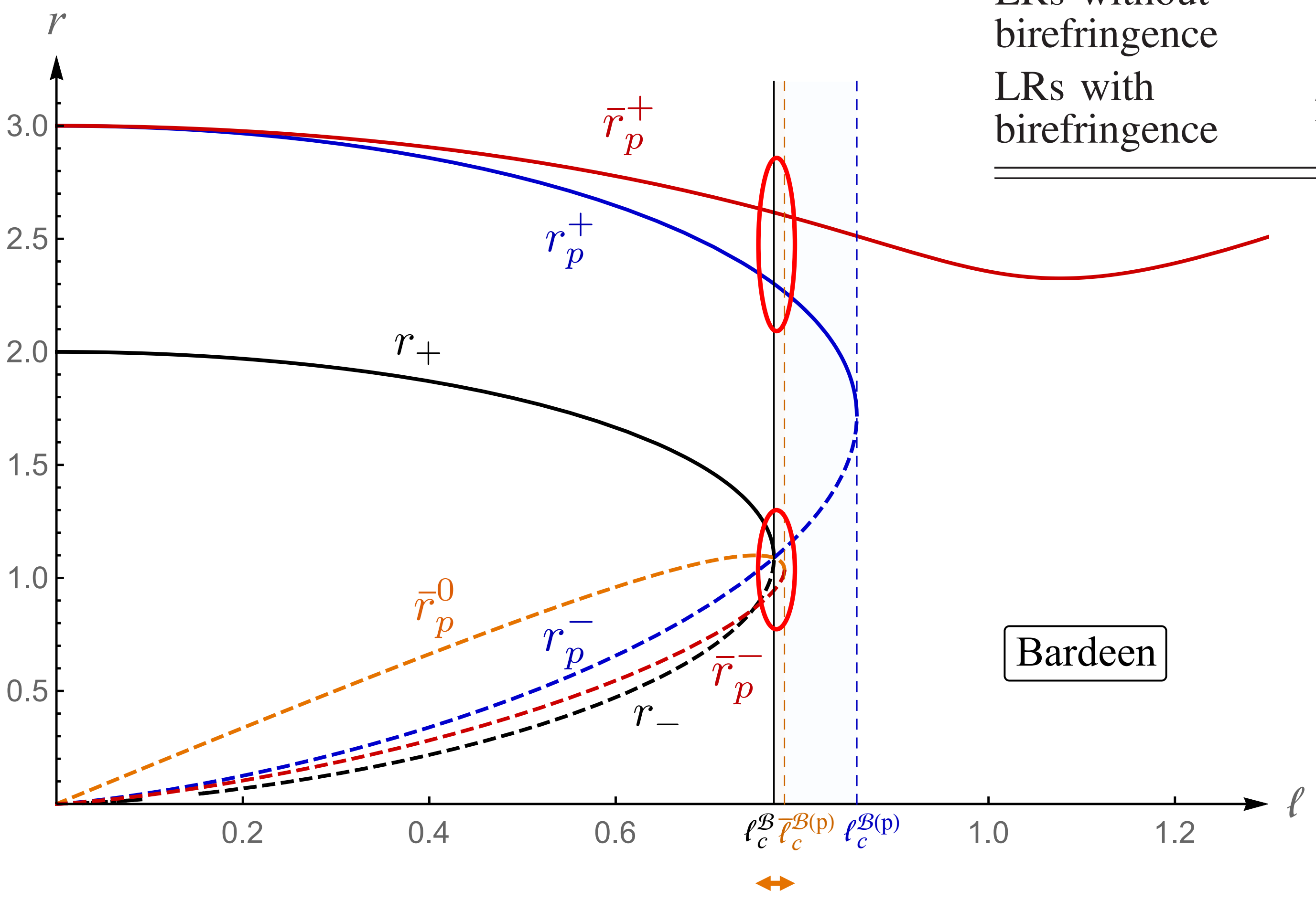
Light Rings: Bardeen Model

	$0 < \ell < \ell_c$	$\ell_c < \ell < \bar{\ell}_c^{(p)}$
Nonsingular UCO type	RBH	Horizonless
LRs without birefringence	1	2
LRs with birefringence	2	5



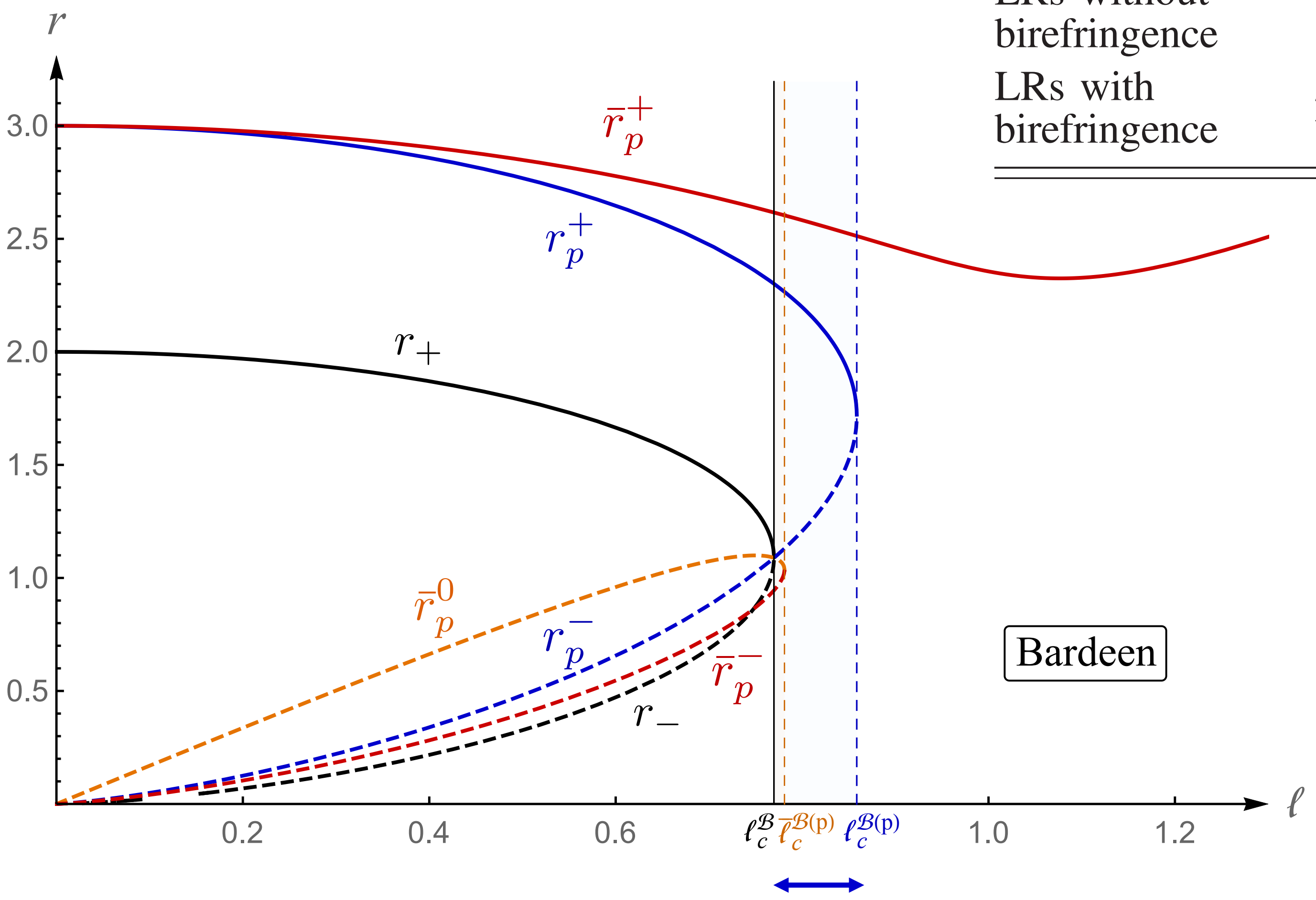
Light Rings: Bardeen Model

	$0 < \ell < \ell_c$	$\ell_c < \ell < \bar{\ell}_c^{(p)}$
Nonsingular UCO type	RBH	Horizonless
LRs without birefringence	1	2
LRs with birefringence	2	5



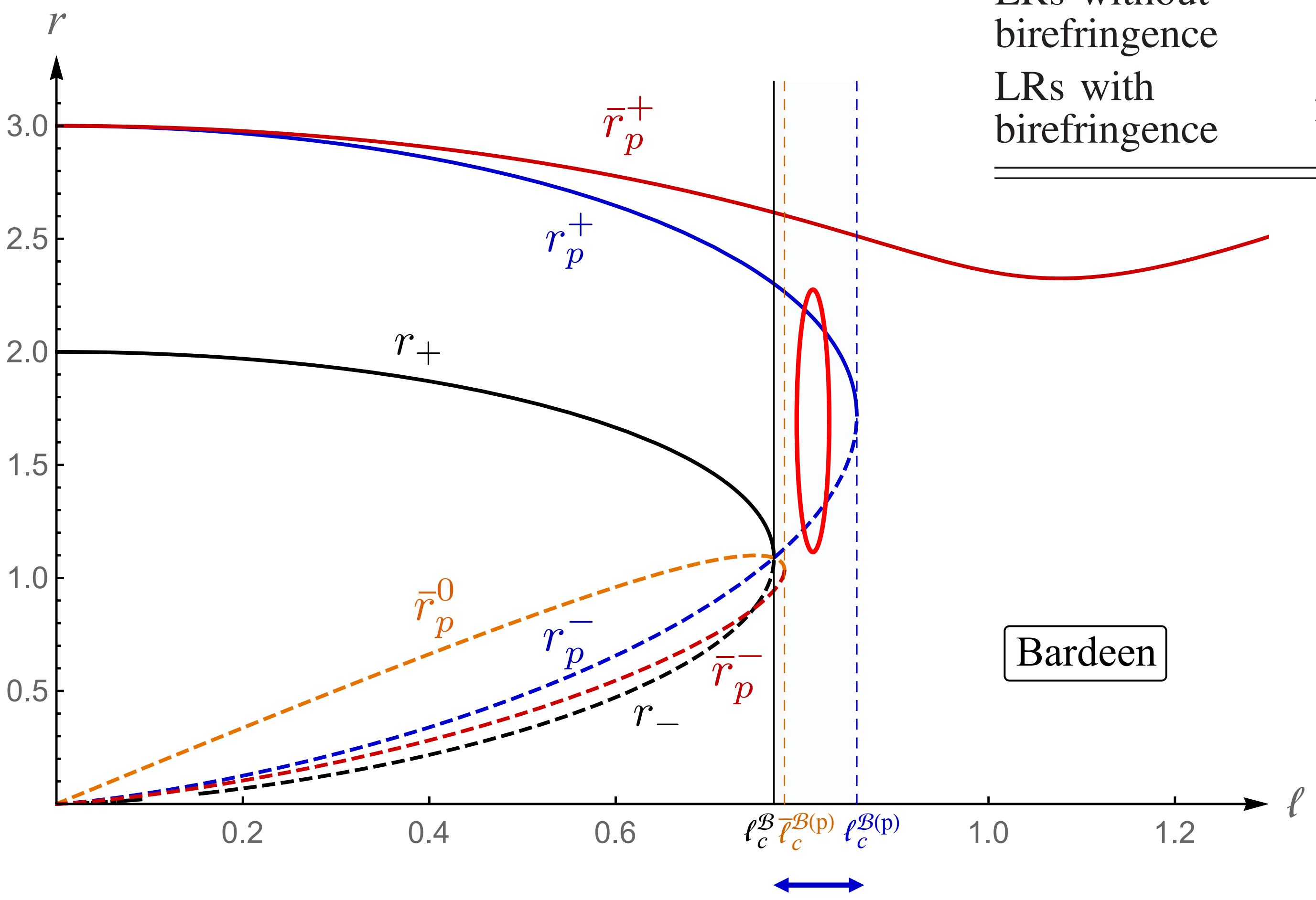
Light Rings: Bardeen Model

	$0<\ell<\ell_c$	$\ell_c<\ell<\bar{\ell}_c^{(p)}$	$\bar{\ell}_c^{(p)}<\ell<\ell_c^{(p)}$
Nonsingular UCO type	RBH	Horizonless	Horizonless
LRs without birefringence	1	2	2
LRs with birefringence	2	5	3

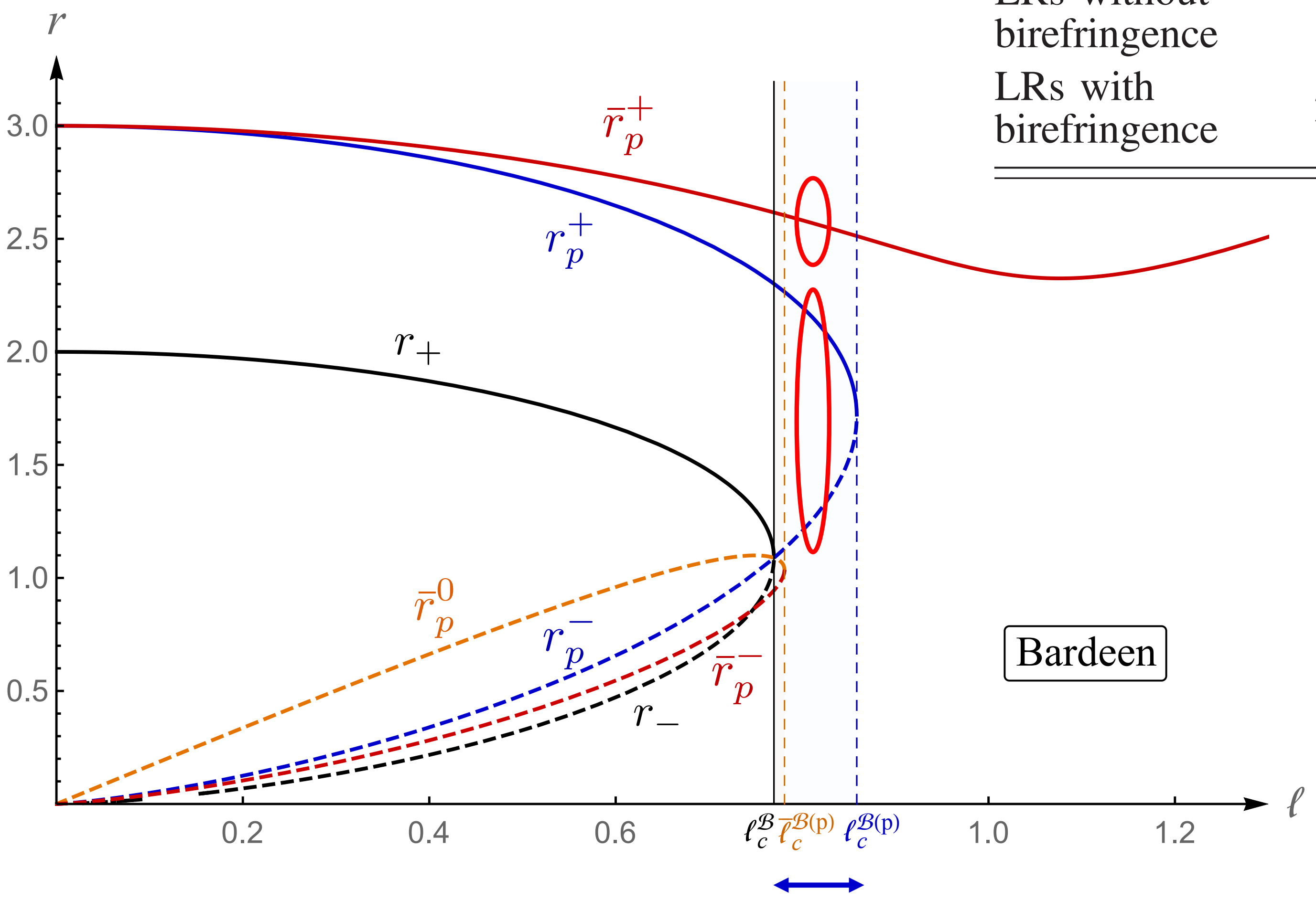


Light Rings: Bardeen Model

	$0 < \ell < \ell_c$	$\ell_c < \ell < \overline{\ell}_c^{(p)}$	$\overline{\ell}_c^{(p)} < \ell < \ell_c^{(p)}$
Nonsingular UCO type	RBH	Horizonless	Horizonless
LRs without birefringence	1	2	2
LRs with birefringence	2	5	3



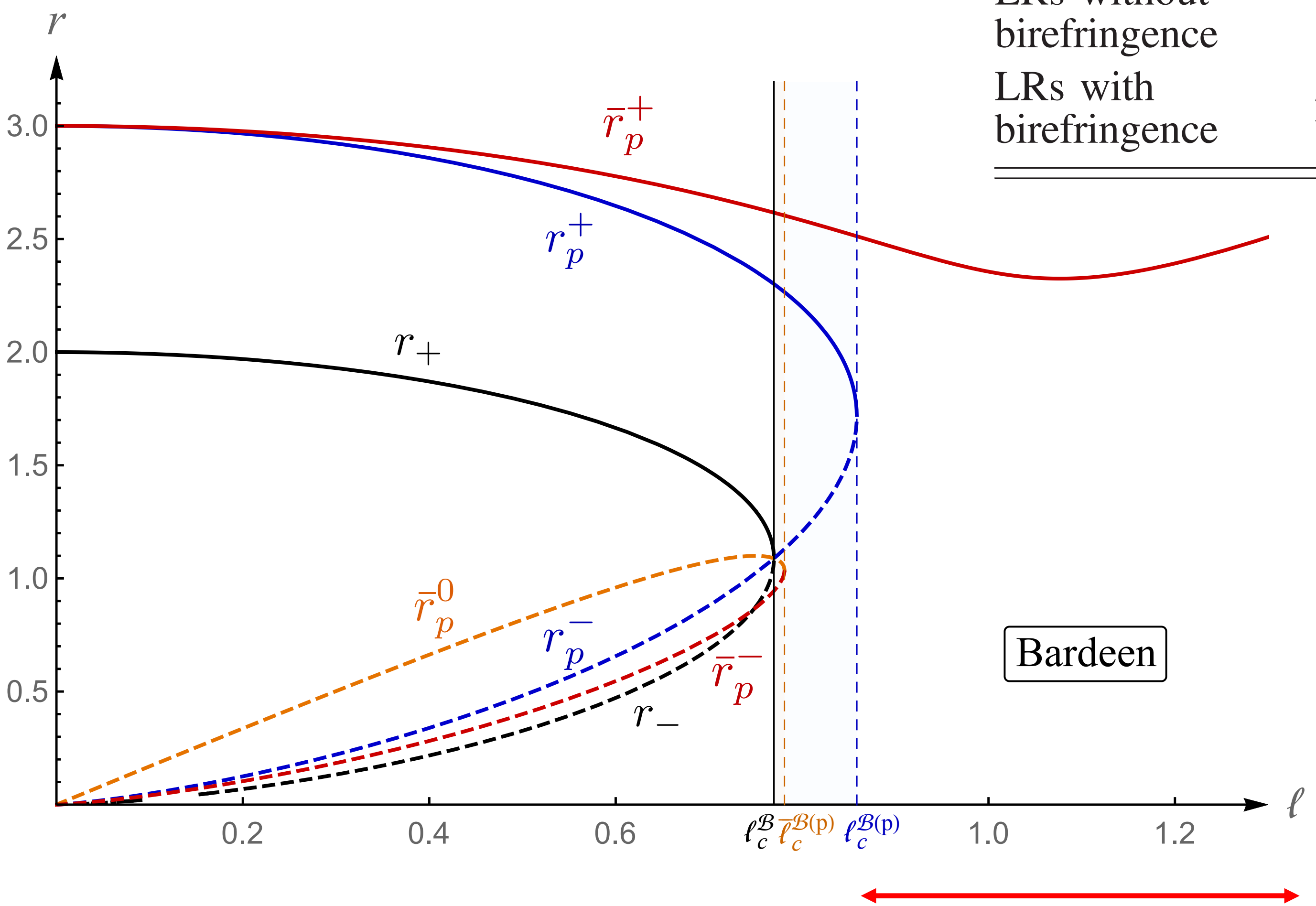
Light Rings: Bardeen Model



	$0<\ell<\ell_c$	$\ell_c<\ell<\bar{\ell}_c^{(p)}$	$\bar{\ell}_c^{(p)}<\ell<\ell_c^{(p)}$
Nonsingular UCO type	RBH	Horizonless	Horizonless
LRs without birefringence	1	2	2
LRs with birefringence	2	5	3

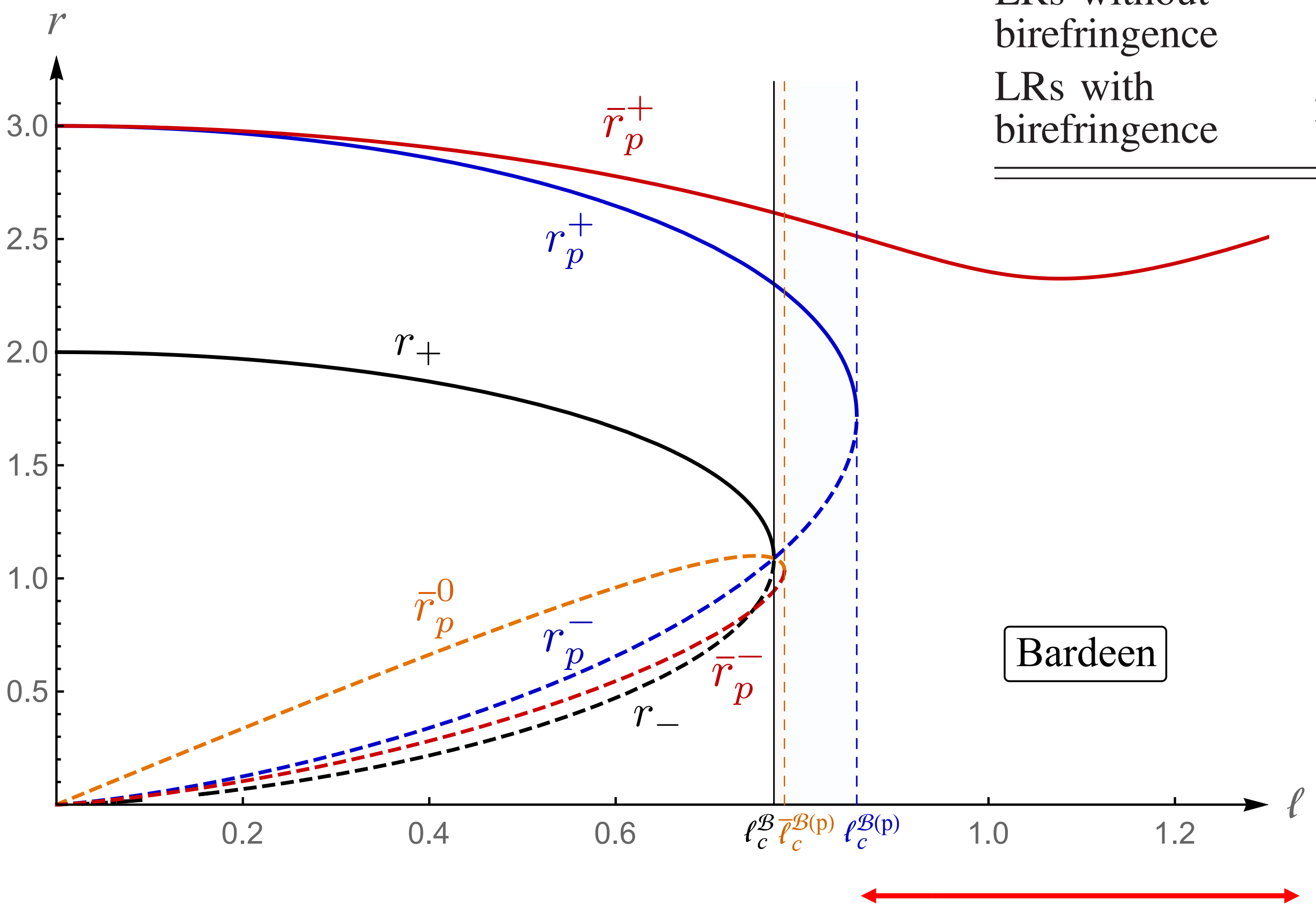
Light Rings: Bardeen Model

	$0 < \ell < \ell_c$	$\ell_c < \ell < \overline{\ell}_c^{(p)}$	$\overline{\ell}_c^{(p)} < \ell < \ell_c^{(p)}$	$\ell > \ell_c^{(p)}$
Nonsingular UCO type	RBH	Horizonless	Horizonless	Horizonless
LRs without birefringence	1	2	2	0
LRs with birefringence	2	5	3	1



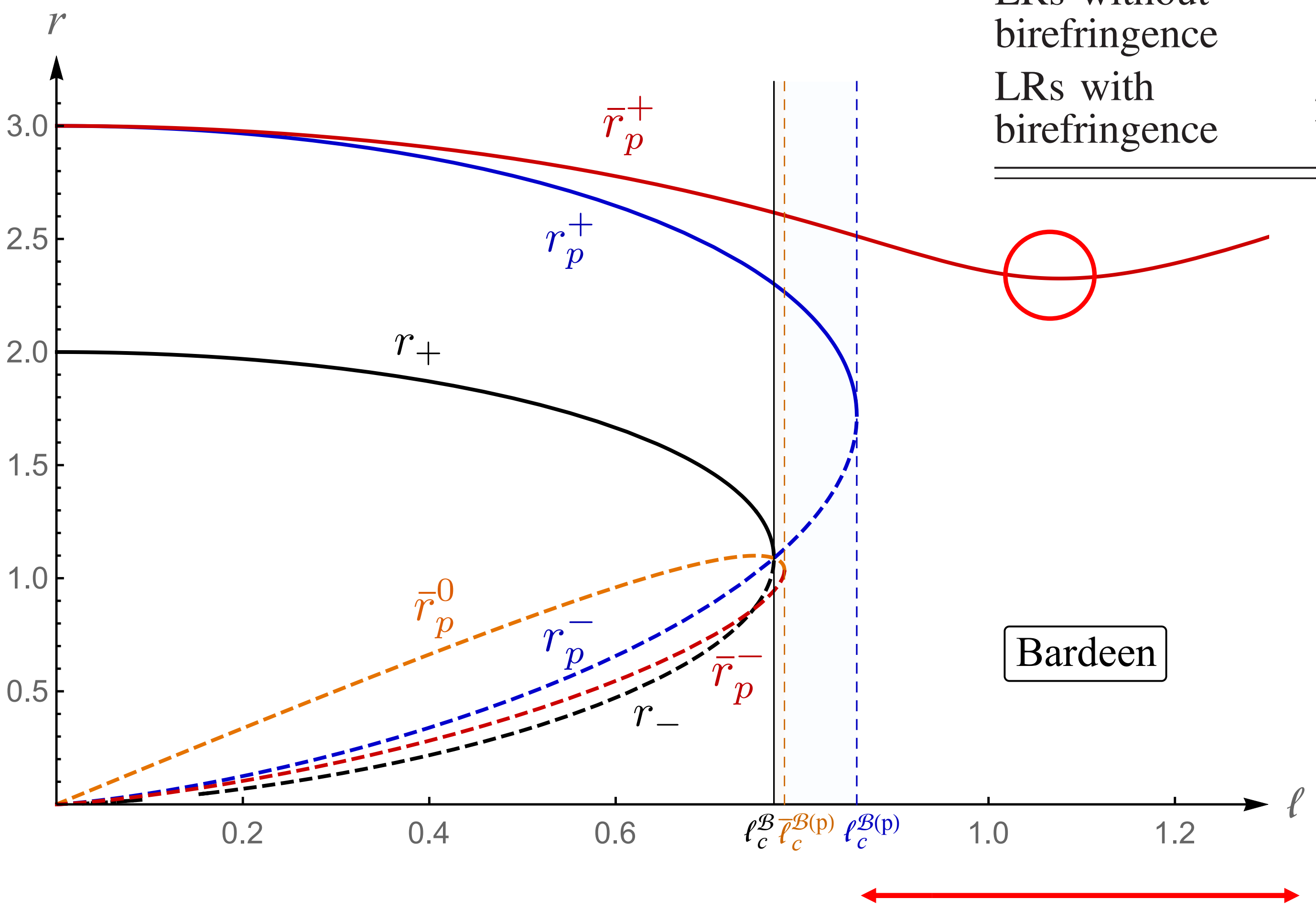
Light Rings: Bardeen Model

	$0 < \ell < \ell_c$	$\ell_c < \ell < \overline{\ell}_c^{(p)}$	$\overline{\ell}_c^{(p)} < \ell < \ell_c^{(p)}$	$\ell > \ell_c^{(p)}$
Nonsingular UCO type	RBH	Horizonless	Horizonless	Horizonless
LRs without birefringence	1	2	2	0
LRs with birefringence	2	5	3	1

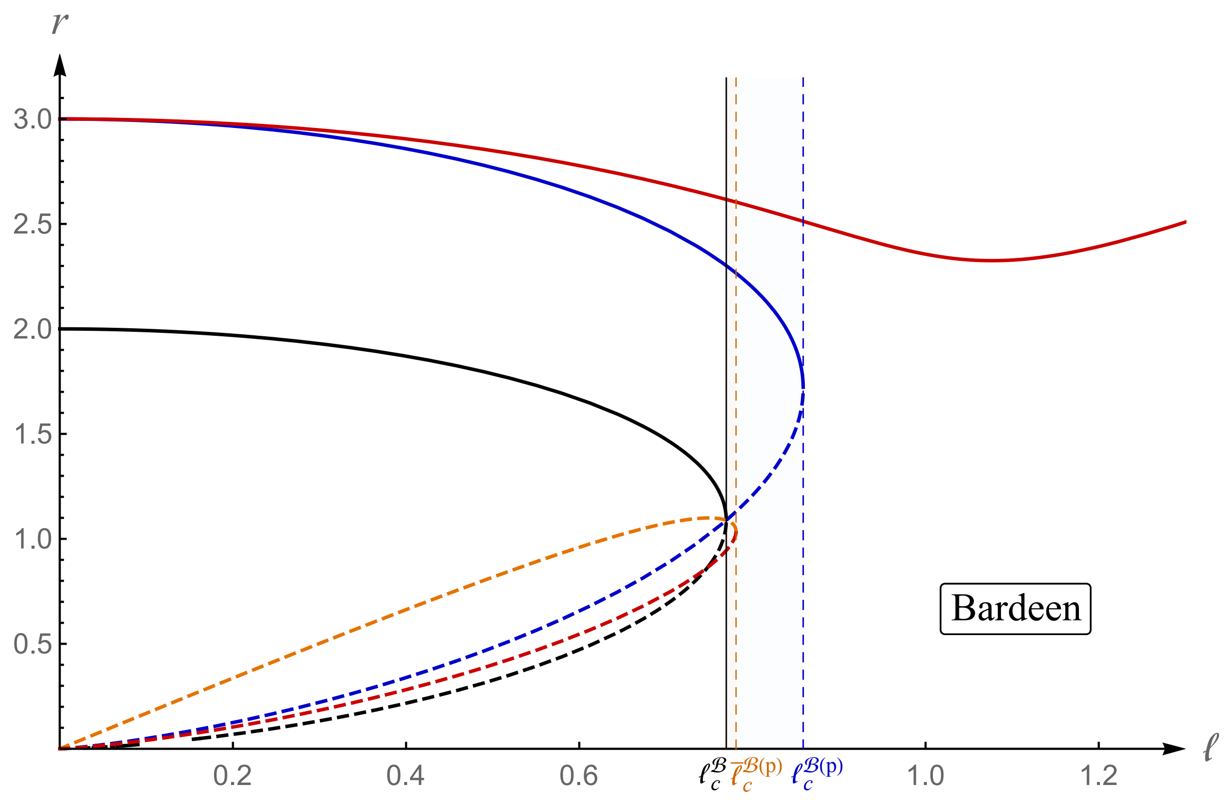


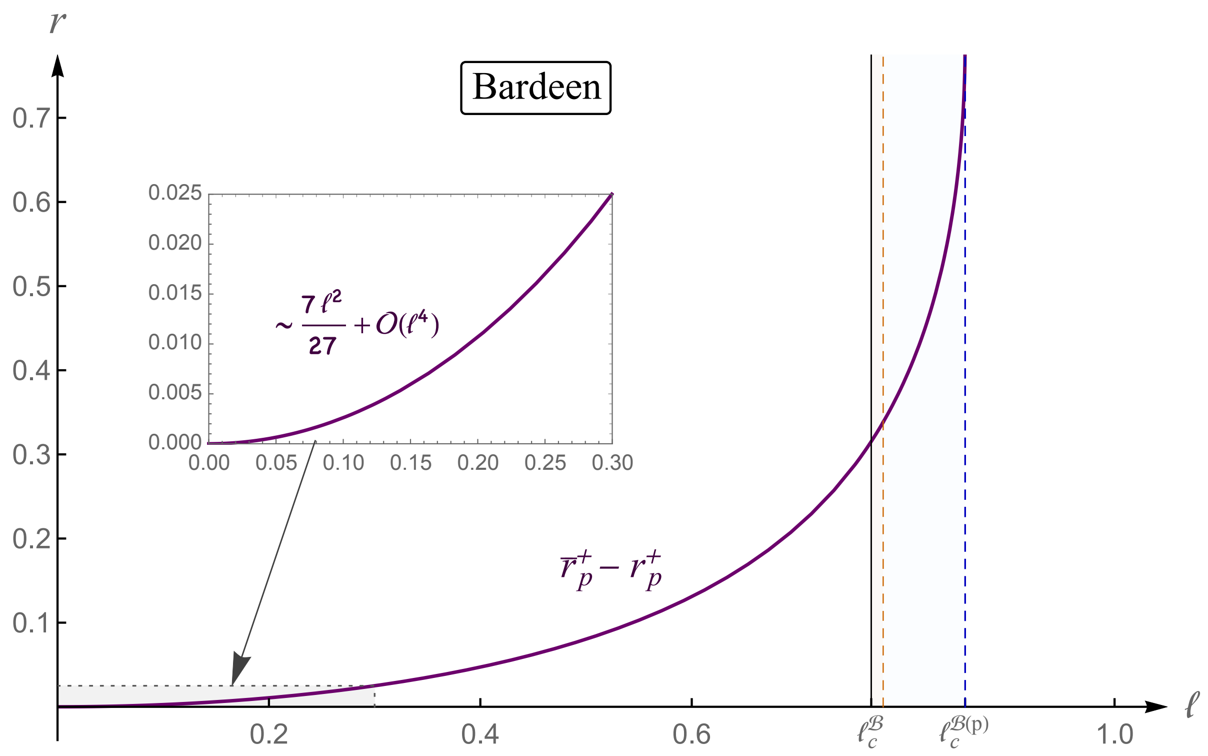
Light Rings: Bardeen Model

	$0 < \ell < \ell_c$	$\ell_c < \ell < \overline{\ell}_c^{(p)}$	$\overline{\ell}_c^{(p)} < \ell < \ell_c^{(p)}$	$\ell > \ell_c^{(p)}$
Nonsingular UCO type	RBH	Horizonless	Horizonless	Horizonless
LRs without birefringence	1	2	2	0
LRs with birefringence	2	5	3	1

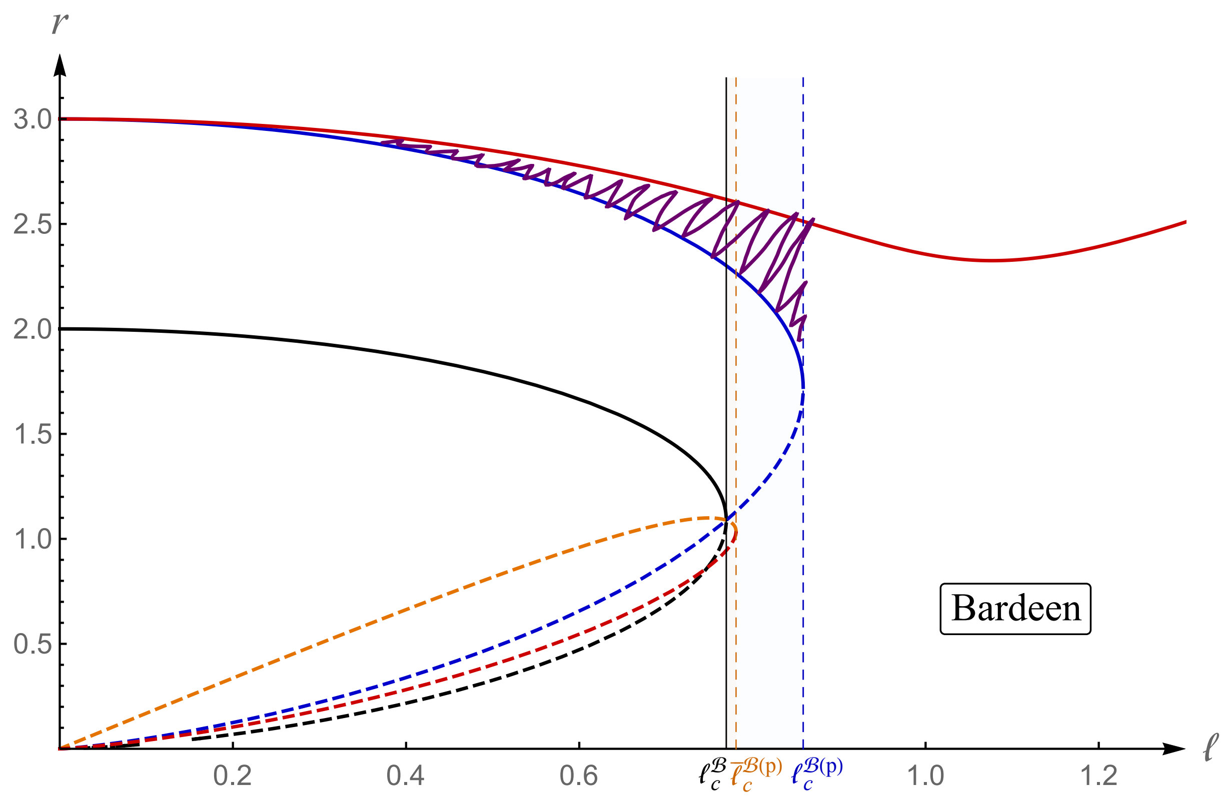


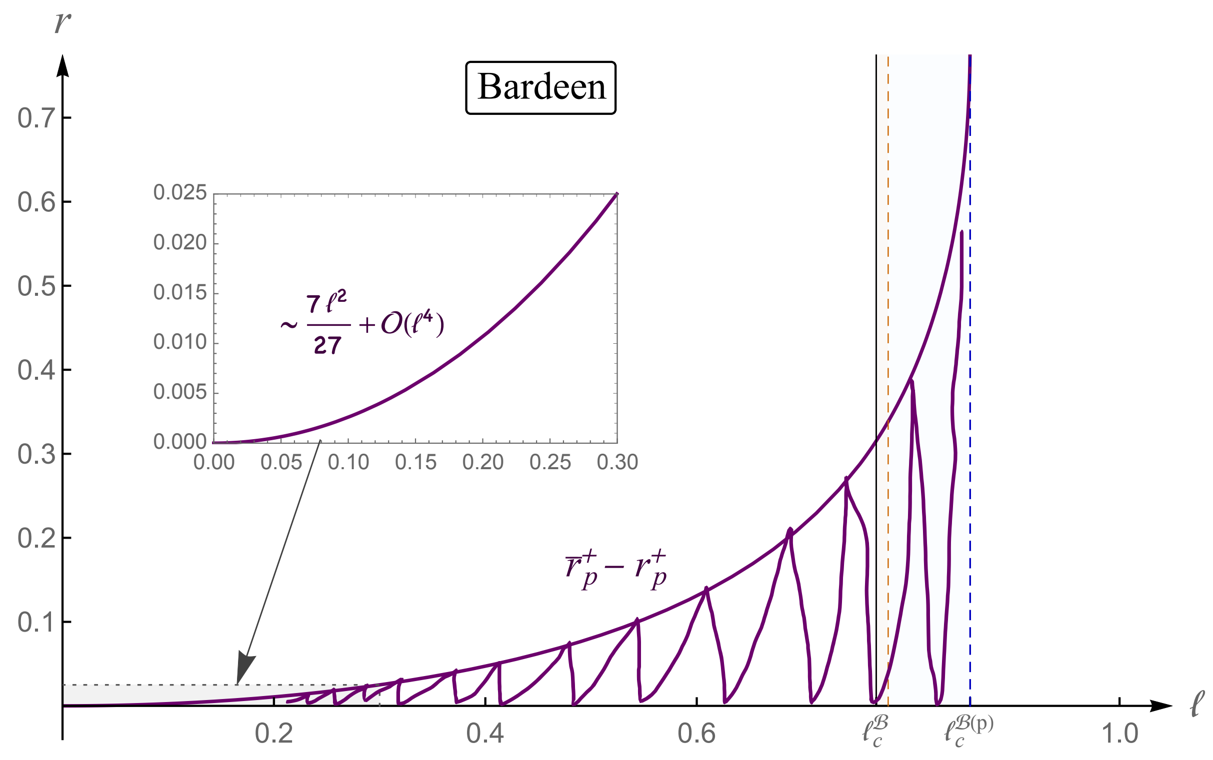
Light Rings: Bardeen Model



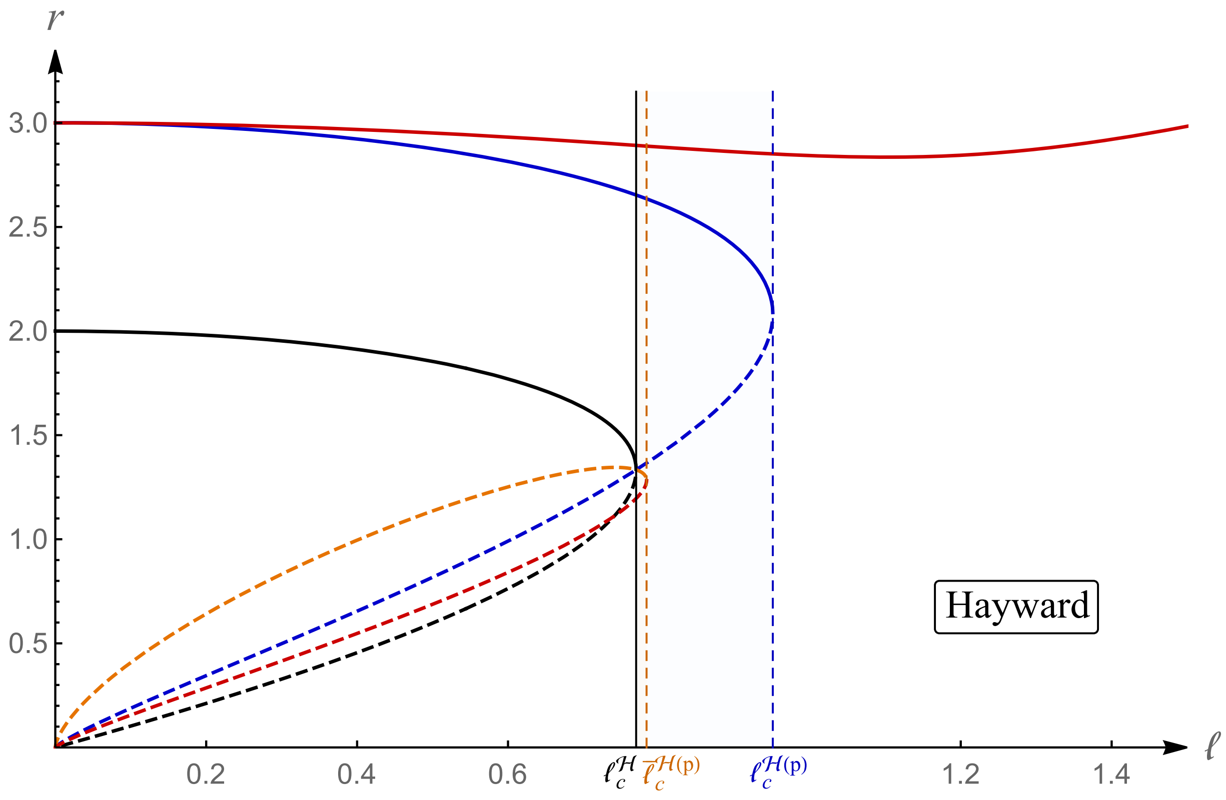


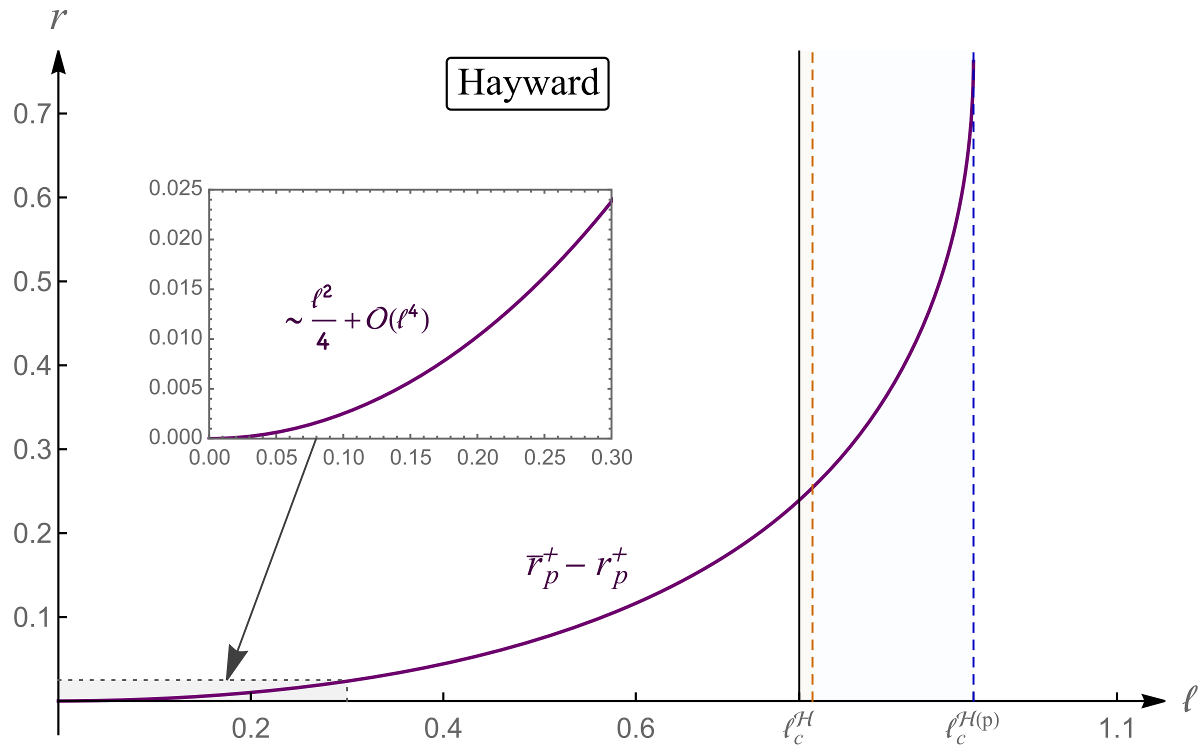
Light Rings: Bardeen Model



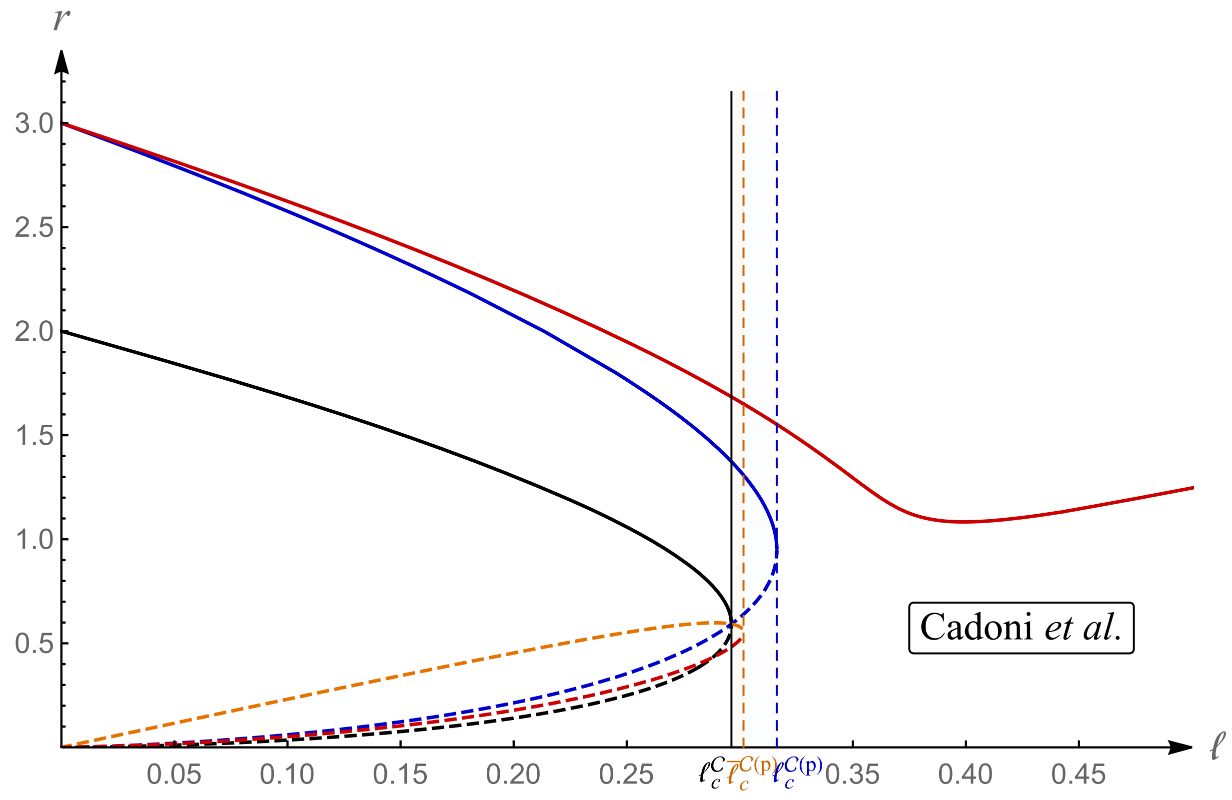


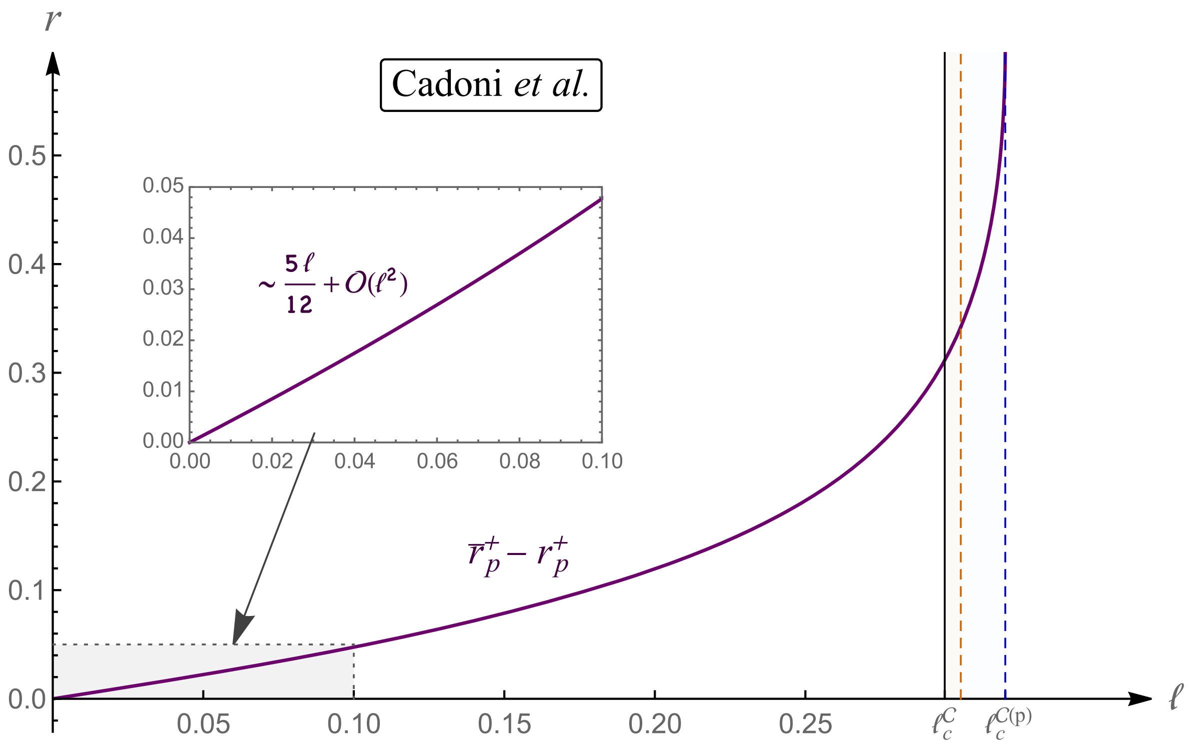
Light Rings: Hayward Model



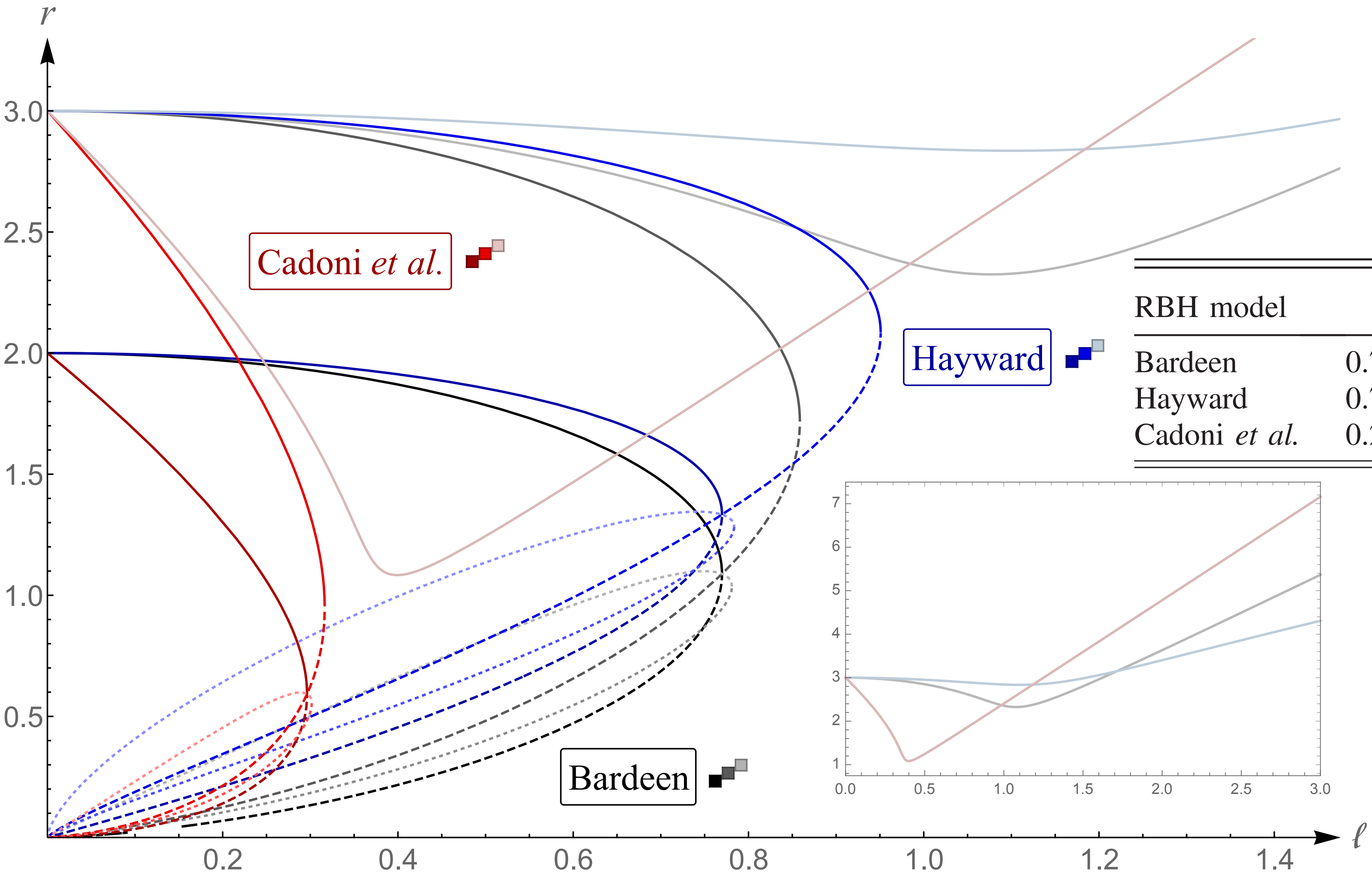


Light Rings: Cadoni *et al.* Model





Light Rings: Summary



RBH model	ℓ_c	$\overline{\ell}_c^{(p)}$	$\ell_c^{(p)}$	$\overline{\ell}^{+}_{\min}$
Bardeen	$0.7698M$	$0.7811M$	$0.8587M$	$1.0766M$
Hayward	$0.7698M$	$0.7836M$	$0.9509M$	$1.1004M$
Cadoni *et al.*	$0.2963M$	$0.3016M$	$0.3164M$	$0.3991M$

Phase velocity

Wave vector $k_\mu = (-\omega, \sqrt{g_{rr}}|\vec{k}|\cos\eta, 0, \sqrt{g_{\phi\phi}}|\vec{k}|\sin\eta)$

$|\vec{k}| = \sqrt{g^{ij}k_i k_j}$

$i, j \in \{1, 2, 3\}$

$\eta \in [0, \pi]$

$\eta = 0:$ radial

$\eta = \pi/2:$ circular

Phase velocity $v_{\text{ph}} = \dfrac{\omega}{|\vec{k}|}$

Use null condition $g_{\mu\mu}k^\mu k^\mu = 0 \quad \Rightarrow \quad g^{ii}k_i k_i = -g^{tt}k_t k_t$

$$\Rightarrow \quad v_{\text{ph}} = \sqrt{-g_{tt}} = \sqrt{f(r)}$$

Independent of η !

$$\bar{v}_{\text{ph}}(\eta) = \sqrt{f(r)\left(1 + \frac{2\mathcal{F}\mathcal{L}_{\mathcal{F}\mathcal{F}}}{\mathcal{L}_{\mathcal{F}}}\sin^2\eta\right)}$$

For $\eta = 0$: $\bar{v}_{\text{ph}} = v_{\text{ph}}$

η-dependence of phase velocity

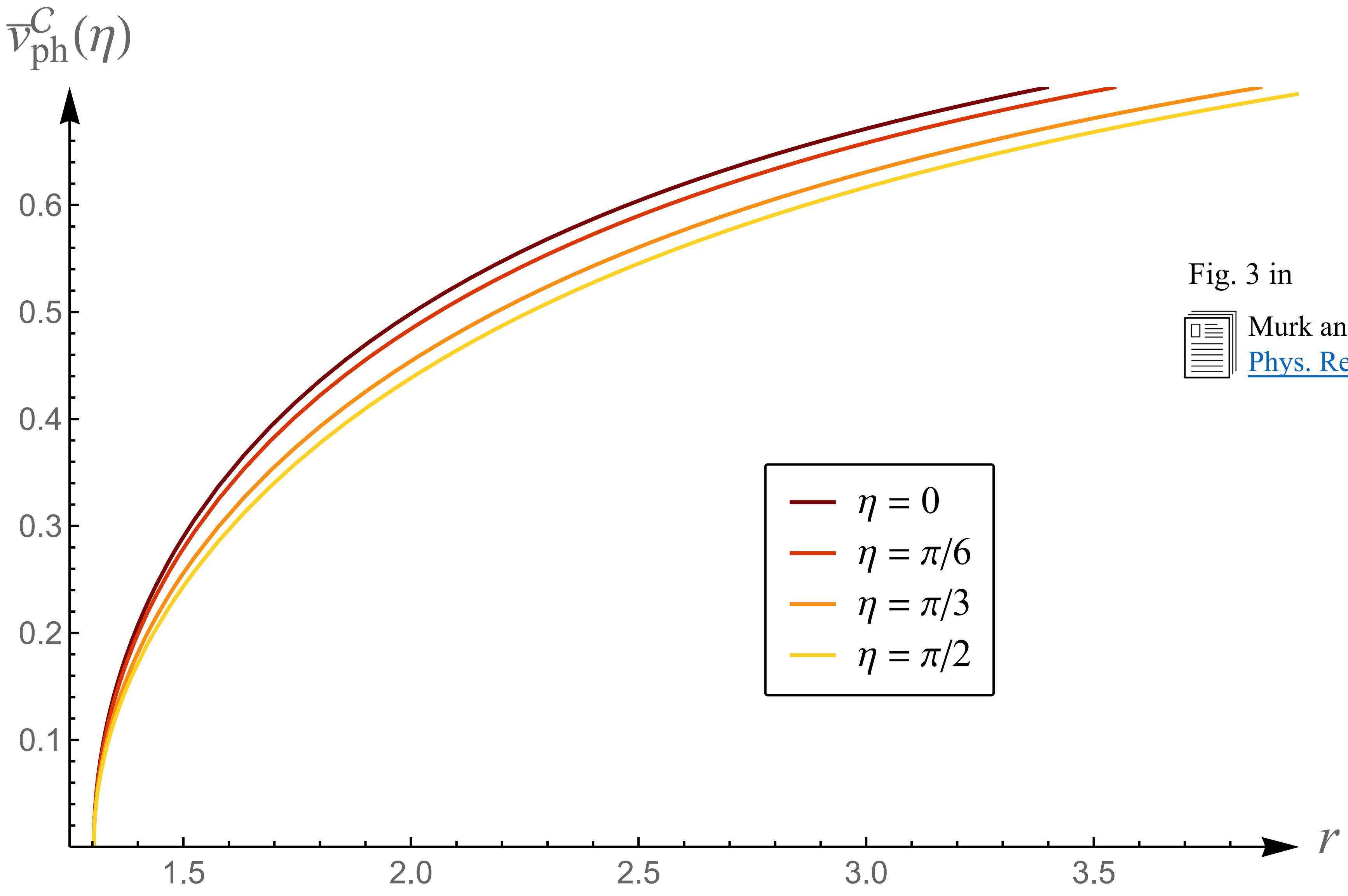


Fig. 3 in

Murk and Soranidis
Phys. Rev. D **110**, 044064 (2024)

Phase velocity: background vs. effective metric

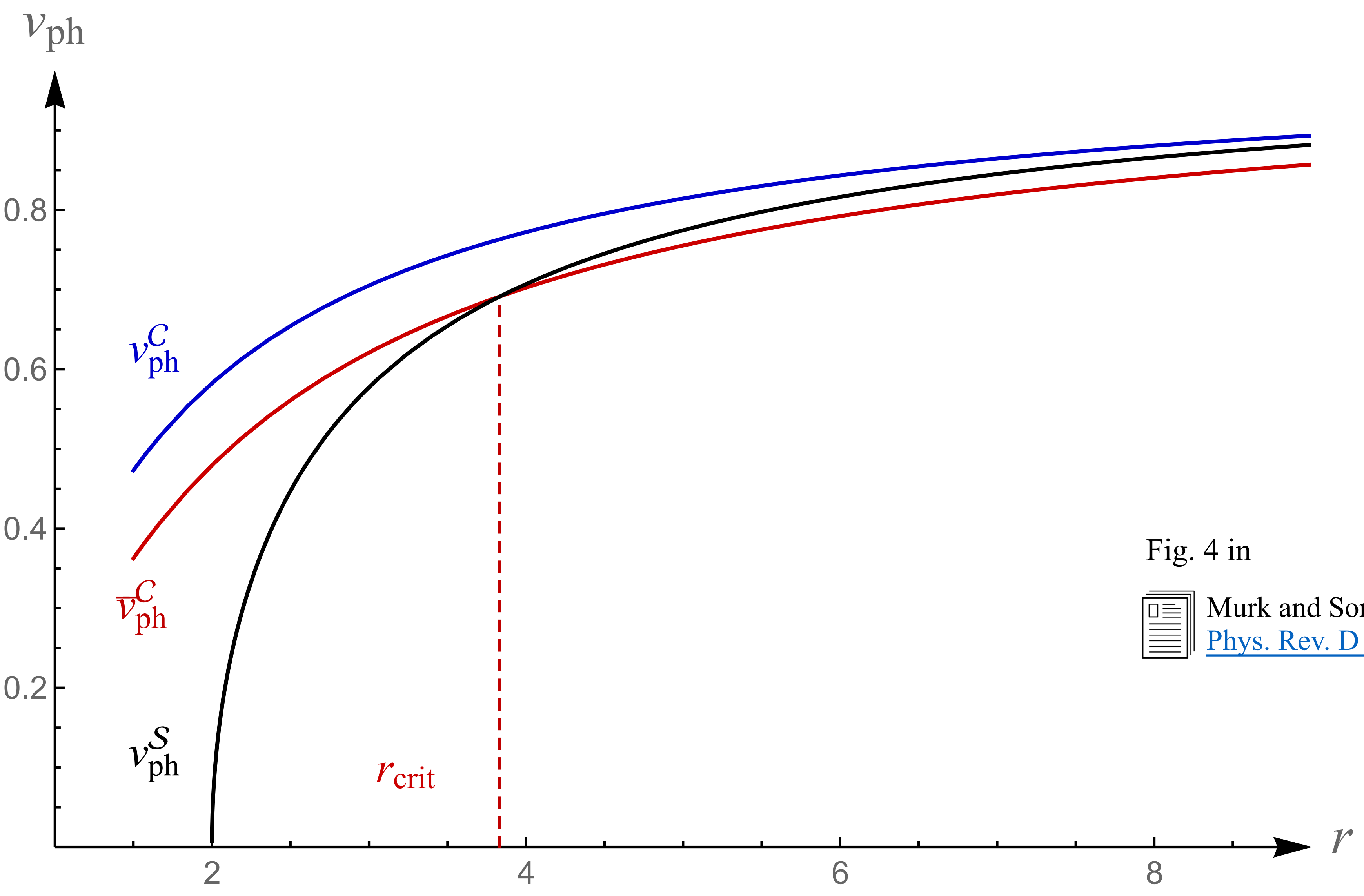


Fig. 4 in

Murk and Soranidis
[Phys. Rev. D **110**, 044064 (2024)]

Causality considerations

$$\overset{?}{\lessgtr} 0$$

$$k^2 = g_{\mu\nu}k^\mu k^\nu = -g_{tt}\frac{2\mathcal{F}\mathcal{L}_{\mathcal{F}\mathcal{F}}}{\mathcal{L}_{\mathcal{F}}}(k^t)^2$$

$$> 0 \qquad > 0 \qquad > 0$$

➡ $\boxed{\text{Condition for causal propagation:} \quad \mathcal{L}_{\mathcal{F}\mathcal{F}} < 0}$

Ensures that what moves on a null geodesic in the effective geometry is timelike in the background geometry, thereby preserving the causal character of the theory.

Result: No acausal regions in NED theories that conform to the Maxwell weak-field limit (Cadoni *et al.*). Otherwise (Bardeen, Hayward), the light cone of the background geometry is fully contained within the light cone of the effective geometry, which indicates the presence of spacetime regions where superluminal motion is possible (provided that the relevant trajectories exist).

BACKUP SLIDES

APPENDIX A: BIREFRINGENCE

Following the argumentation of Ref. [36], we rederive the phenomenon of birefringence for convenience. As in Sec. II, we restrict our considerations to NED Langrangian densities $\mathcal{L}(\mathcal{F},\mathcal{G}) \equiv \mathcal{L}(\mathcal{F})$. The derivation is based on the assumption that the electromagnetic field $\tilde{F}_{\mu\nu}$ of the UCO is much stronger than that of photons $\Phi_{\mu\nu}$. For simplicity, we work in Minkowski spacetime here, but an analogous derivation can be performed in generic curved spacetimes by considering covariant derivatives instead of partial derivatives in what follows. The total electromagnetic field is given by

$$F_{\mu\nu} = \tilde{F}_{\mu\nu} + \Phi_{\mu\nu}. \tag{A1}$$

Defining

$$h^{\mu\nu} := \mathcal{L}_{\mathcal{F}} F^{\mu\nu}, \tag{A2}$$

the leading terms in the series expansion about $\Phi_{\mu\nu}$ are

$$h^{\mu\nu} = h^{\mu\nu}|_{\Phi=0} + \frac{\partial h^{\mu\nu}}{\partial F_{\rho\sigma}}\bigg|_{F=\tilde{F}} \Phi_{\rho\sigma} + \mathcal{O}(\Phi^2), \tag{A3}$$

where indices have been omitted in the $|_\bullet$ subscripts and order $\mathcal{O}(\bullet)$ here and in what follows to avoid clutter. Substituting Eq. (A3) into Eq. (A2) results in the linearized equations

$$\partial_\mu h^{\mu\nu} = \frac{\partial h^{\mu\nu}}{\partial F_{\rho\sigma}}\bigg|_{F=\tilde{F}} \partial_\mu \Phi_{\rho\sigma} + \mathcal{O}(\Phi) = 0. \tag{A4}$$

We are looking for solutions $\Phi_{\rho\sigma}$ of the form

$$\Phi_{\rho\sigma}(x) = \epsilon_{\rho\sigma}(k) e^{-ikx}, \tag{A5}$$

where

$$\epsilon_{\rho\sigma} := k_\rho \epsilon_\sigma(k) - k_\sigma \epsilon_\rho(k) \tag{A6}$$

denotes the antisymmetric polarization tensor, k_ρ the propagation vector, and ϵ_ρ the polarization vector [76]. Substituting the partial derivative

$$\partial_\mu \Phi_{\rho\sigma} = -ik_\mu \epsilon_{\rho\sigma}(k) e^{-ikx} \tag{A7}$$

into Eq. (A4) results in

$$\frac{\partial h^{\mu\nu}}{\partial F_{\rho\sigma}}\bigg|_{F=\tilde{F}} (-ik_\mu \epsilon_{\rho\sigma}(k) e^{-ikx}) = 0, \tag{A8}$$

and using the definition of the antisymmetric polarization tensor Eq. (A6), we have

$$\frac{\partial h^{\mu\nu}}{\partial F_{\rho\sigma}}\bigg|_{F=\tilde{F}} k_\mu (k_\rho \epsilon_\sigma - k_\sigma \epsilon_\rho) = 0. \tag{A9}$$

Finally, using the antisymmetry of the electromagnetic field tensor $F_{\rho\sigma}$ [cf. Eq. (2.1)], we arrive at the relation

$$\frac{\partial h^{\mu\nu}}{\partial F_{\rho\sigma}}\bigg|_{F=\tilde{F}} k_\mu k_\rho \epsilon_\sigma = 0. \tag{A10}$$

The partial derivative is calculated as

$$\frac{\partial h^{\mu\nu}}{\partial F_{\rho\sigma}} \overset{(A2)}{=} \frac{\partial}{\partial F_{\rho\sigma}} (\mathcal{L}_{\mathcal{F}} F^{\mu\nu}) = \mathcal{L}_{\mathcal{F}\mathcal{F}} \frac{\partial \mathcal{F}}{\partial F_{\rho\sigma}} F^{\mu\nu} + \mathcal{L}_{\mathcal{F}} \frac{\partial F^{\mu\nu}}{\partial F_{\rho\sigma}}, \tag{A11}$$

where

$$\frac{\partial F^{\mu\nu}}{\partial F_{\rho\sigma}} = \frac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\nu\rho}\eta^{\mu\sigma}) \tag{A12}$$

due to the antisymmetry property [cf. Eq. (2.1)]. The derivative of the field strength $\mathcal{F}$ with respect to $F_{\rho\sigma}$ is calculated using this relation and results in

$$\begin{aligned}\frac{\partial \mathcal{F}}{\partial F_{\rho\sigma}} &= \frac{\partial}{\partial F_{\rho\sigma}}(F_{\alpha\beta}F^{\alpha\beta}) \\ &= \frac{\partial}{\partial F_{\rho\sigma}}(\eta_{\alpha\chi}\eta_{\beta\lambda}F^{\alpha\beta}F^{\chi\lambda}) = 2F^{\rho\sigma}.\end{aligned} \tag{A13}$$

Substitution of Eqs. (A12) and (A13) into Eq. (A11) and subsequently into Eq. (A10) results in

$$2\mathcal{L}_{\mathcal{F}\mathcal{F}}(F^{\rho\sigma}k_\rho)(F^{\mu\nu}k_\mu)\epsilon_\sigma + \frac{1}{2}\mathcal{L}_{\mathcal{F}}(\eta^{\nu\sigma}k^2 - k^\nu k^\sigma)\epsilon_\sigma = 0. \tag{A14}$$

It is useful to define the four-vectors

$$a^\mu := F^{\mu\nu}k_\nu, \qquad {}^\star a^\mu := {}^\star F^{\mu\nu}k_\nu, \qquad b^\mu := F^{\mu\nu}a_\nu. \tag{A15}$$

Equation (A14) can then be rewritten as

$$[\mathcal{L}_{\mathcal{F}}(\eta^{\nu\sigma}k^2 - k^\nu k^\sigma) + 4\mathcal{L}_{\mathcal{F}\mathcal{F}}a^\nu a^\sigma]\epsilon_\sigma = 0. \tag{A16}$$

Defining

$$M^{\nu\sigma} := \mathcal{L}_{\mathcal{F}}(\eta^{\nu\sigma}k^2 - k^\nu k^\sigma) + 4\mathcal{L}_{\mathcal{F}\mathcal{F}}a^\nu a^\sigma, \tag{A17}$$

Equation (A16) is given by

$$M^{\nu\sigma}\epsilon_\sigma = 0. \tag{A18}$$

To determine the polarization vector ϵ_σ, we express it in terms of four linearly independent vectors a^μ, ${}^\star a^\mu$, k^μ, and b^μ, i.e.,

$$\epsilon_\sigma = c_1 a_\sigma + c_2 {}^\star a_\sigma + c_3 k_\sigma + c_4 b_\sigma. \tag{A19}$$

Equation (A18) is then given by

$$c_1 M^{\nu\sigma}a_\sigma + c_2 M^{\nu\sigma\star}a_\sigma + c_3 M^{\nu\sigma}k_\sigma + c_4 M^{\nu\sigma}b_\sigma = 0. \tag{A20}$$

Using $k^\sigma a_\sigma = k^\sigma F_{\sigma\alpha}k^\alpha = 0$ (due to antisymmetry), the first term is given by

$$M^{\nu\sigma}a_\sigma = (\mathcal{L}_{\mathcal{F}}k^2 + 4\mathcal{L}_{\mathcal{F}\mathcal{F}}a^2)a^\nu, \tag{A21}$$

where $a^2 := a_\mu a^\mu$. Similarly, $k^{\sigma\star}a_\sigma = k^{\sigma\star}F_{\sigma\nu}k^\nu = 0$ (due to antisymmetry), and the second term is given by

$$M^{\nu\sigma\star}a_\sigma = \mathcal{L}_\mathcal{F} k^{2\star} a^\nu + 4\mathcal{L}_{\mathcal{F}\mathcal{F}}(a^{\sigma\star}a_\sigma)a^\nu. \tag{A22}$$

Analogously, the third and fourth term are given by

$$M^{\nu\sigma}k_\sigma = \mathcal{L}_\mathcal{F}(k^2k^\nu - k^2k^\nu) + 4\mathcal{L}_{\mathcal{F}\mathcal{F}}(a^\sigma k_\sigma)a^\nu = 0, \tag{A23}$$

$$M^{\nu\sigma}b_\sigma = \mathcal{L}_\mathcal{F} k^2 b^\nu - \mathcal{L}_\mathcal{F}(k^\sigma b_\sigma)k^\nu, \tag{A24}$$

respectively, where we have used $a^\sigma k_\sigma = 0$ and $a^\sigma b_\sigma = a^\sigma F_{\sigma\alpha}a^\alpha = 0$ (due to antisymmetry). Substitution of Eqs. (A21)–(A24) into Eq. (A20) yields

$$\begin{aligned}
&[c_1(\mathcal{L}_\mathcal{F}k^2 + 4\mathcal{L}_{\mathcal{F}\mathcal{F}}a^2) + c_2 4\mathcal{L}_{\mathcal{F}\mathcal{F}}(a^{\sigma\star}a_\sigma)]a^\nu \\
&\quad + c_2(\mathcal{L}_\mathcal{F}k^2)^\star a^\nu - c_4[\mathcal{L}_\mathcal{F}(k^\sigma b_\sigma)]k^\nu + c_4(\mathcal{L}_\mathcal{F}k^2)b^\nu = 0.
\end{aligned} \tag{A25}$$

Since the four-vectors a^μ, ${}^\star a^\mu$, k^μ, and b^μ are linearly independent, the vanishing of $M^{\nu\sigma}\epsilon_\sigma$ implies that the coefficients must vanish. We note that

$$k^\sigma b_\sigma = k^\sigma F_{\sigma\alpha}a^\alpha = -(F_{\alpha\sigma}k^\sigma)a^\alpha = -a_\alpha a^\alpha = -a^2, \tag{A26}$$

and

$$\begin{aligned}
a^{\sigma\star}a_\sigma &= F^{\sigma\beta}k_\beta {}^\star F_{\sigma\alpha}k^\alpha = \eta^{\alpha\lambda}(-\mathcal{G}\eta^\beta_\alpha)k_\beta k_\lambda \\
&= -\mathcal{G}\eta^{\beta\lambda}k_\sigma k_\lambda = -\mathcal{G}k^2,
\end{aligned} \tag{A27}$$

where we have used the property $F^{\sigma\beta\star}F_{\sigma\alpha} = -\mathcal{G}\eta^\beta_\alpha$ with $\mathcal{G}$ as defined in Eq. (2.2).

Since the last two terms in Eq. (A25) cannot vanish simultaneously unless $c_4 = 0$, we are left with a system of equations for the coefficients c_1 and c_2.[12] For $k^2 = 0$, we have

$$c_1(4\mathcal{L}_{\mathcal{F}\mathcal{F}}a^2) = 0, \tag{A28}$$

and an arbitrary c_2, which necessarily leads to $c_1 = 0$, and thus the polarization vector is

$$\epsilon^\mu = c_2{}^\star a^\mu. \tag{A29}$$

This corresponds to the photon polarization mode that moves on null geodesics in the background geometry. For $k^2 \neq 0$ on the other hand, we find

$$c_1(\mathcal{L}_\mathcal{F}k^2 + 4\mathcal{L}_{\mathcal{F}\mathcal{F}}a^2) = 0. \tag{A30}$$

To find a nontrivial (i.e., $c_1 \neq 0$) solution to the eigenvalue problem $M^{\nu\sigma}\epsilon_\sigma = 0$, we solve

[12]Note that the coefficient c_3 does not contribute to the polarization as evident from Eq. (A25). This may also be seen from Eqs. (A5) and (A6): if $\epsilon^\mu \propto k^\mu$, then the polarization tensor $\epsilon_{\rho\sigma}$ and its corresponding field vanishes. Physically, this implies that the polarization vector is always perpendicular to the propagation direction. Hence no part of the polarization vector lies in the propagation direction and thus c_3 does not contribute.

$$\mathcal{L}_{\mathcal{F}}k^2 + 4\mathcal{L}_{\mathcal{F}\mathcal{F}}a^2 = 0 \Rightarrow k^2 + \frac{4\mathcal{L}_{\mathcal{F}\mathcal{F}}}{\mathcal{L}_{\mathcal{F}}}a^2 = 0. \tag{A31}$$

Using the definition of the four-vector a^μ, this equation can be rewritten as

$$k^2 + \frac{4\mathcal{L}_{\mathcal{F}\mathcal{F}}}{\mathcal{L}_{\mathcal{F}}}\eta_{\mu\nu}F^{\mu\alpha}k_\alpha F^{\nu\beta}k_\beta = 0 \tag{A32}$$

$$\Rightarrow \eta^{\alpha\beta}k_\alpha k_\beta + \frac{4\mathcal{L}_{\mathcal{F}\mathcal{F}}}{\mathcal{L}_{\mathcal{F}}}\eta_{\mu\nu}F^{\mu\alpha}F^{\nu\beta}k_\alpha k_\beta = 0 \tag{A33}$$

$$\Rightarrow \left(\eta^{\alpha\beta} + \frac{4\mathcal{L}_{\mathcal{F}\mathcal{F}}}{\mathcal{L}_{\mathcal{F}}}\eta_{\mu\nu}F^{\mu\alpha}F^{\nu\beta}\right)k_\alpha k_\beta = 0. \tag{A34}$$

Consequently, the effective metric tensor is given by

$$\bar{g}^{\alpha\beta} = \eta^{\alpha\beta} + \frac{4\mathcal{L}_{\mathcal{F}\mathcal{F}}}{\mathcal{L}_{\mathcal{F}}}\eta_{\mu\nu}F^{\mu\alpha}F^{\nu\beta}. \tag{A35}$$

Using antisymmetry of the electromagnetic field tensor $F^{\nu\beta} = -F^{\beta\nu}$ and contracting $\eta_{\mu\nu}F^{\beta\nu}$ results in

$$\bar{g}^{\alpha\beta} = \eta^{\alpha\beta} - \frac{4\mathcal{L}_{\mathcal{F}\mathcal{F}}}{\mathcal{L}_{\mathcal{F}}}F^{\alpha}{}_{\mu}F^{\mu\beta}, \tag{A36}$$

which is equivalent to Eq. (2.10). Thus the polarization vector

$$\epsilon^\mu = c_1 a^\mu \tag{A37}$$

corresponds to the photon polarization mode that moves on null geodesics in the effective geometry.

1. Examples

In what follows, we consider circular trajectories. As in Sec. V, we restrict our considerations to the equatorial plane without loss of generality, and thus the wave vector can be written as $k_\mu = (-\omega, 0, 0, k_\phi)$.

a. Magnetic solutions

For purely magnetic solutions [$Q_e = 0 \Rightarrow F_{tr} = 0$], the polarization vector that is null in the background geometry [Eq. (A29)] is given explicitly by

$$\begin{aligned}\epsilon^\mu &= c_2{}^\star a^\mu = \frac{c_2}{2}{}^\star F^{\mu\nu}k_\nu = \frac{c_2}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}k_\nu \\ &= c_2\varepsilon^{\mu\nu\theta\phi}F_{\theta\phi}k_\nu = (0, c_2\omega F_{\theta\phi}, 0, 0),\end{aligned} \tag{A38}$$

corresponding to a polarization in the radial direction and a propagation in the ϕ direction.

For the polarization vector that is null in the effective geometry [Eq. (A37)], we have

$$\begin{aligned}\bar{\epsilon}^\mu &= c_1 a^\mu = c_1 F^{\mu\nu}k_\nu = c_1 F^{\theta\phi}k_\phi \\ &= (0, 0, c_1 F^{\theta\phi}k_\phi, 0),\end{aligned} \tag{A39}$$

corresponding to a polarization in the θ direction and a propagation in the ϕ direction.

b. Electric solutions

For purely electric solutions $[Q_m = 0 \Rightarrow F_{\theta\phi} = 0]$, we find

$$\begin{aligned}\epsilon^\mu &= \frac{c_2}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}k_\nu = c_2\varepsilon^{\mu\nu tr}F_{tr}k_\nu \\ &= (0, 0, c_2 F_{tr}k_\phi, 0),\end{aligned} \tag{A40}$$

and

$$\bar{\epsilon}^\mu = c_1 F^{\mu\nu}k_\nu = (0, c_1\omega F^{tr}, 0, 0), \tag{A41}$$

respectively.

APPENDIX B: SINGULARITY OF THE EFFECTIVE METRIC

The line element described by the covariant effective metric tensor components of Eqs. (3.6)–(3.8) is given by

$$ds^2 = -f_i(r)dt^2 + \frac{dr^2}{f_i(r)} + \frac{r^2}{\bar{h}_i(r)}d\Omega^2, \tag{B1}$$

where the subscript $i \in \{\mathcal{B}, \mathcal{H}, \mathcal{C}\}$ labels functions associated with the Bardeen [Eq. (4.2)], Hayward [Eq. (4.12)], and Cadoni *et al.* [Eq. (4.22)] model, respectively, and

$$\bar{h}_i(r) \coloneqq 1 + \frac{2\mathcal{F}\mathcal{L}_{\mathcal{F}\mathcal{F}}}{\mathcal{L}_{\mathcal{F}}}, \tag{B2}$$

with $\mathcal{L}(\mathcal{F})$ given by Eqs. (4.4), (4.14), and (4.24), respectively. Explicit evaluation yields [cf. Eqs. (4.9), (4.10), (4.19), (4.20), (4.29), and (4.30)]

$$\bar{h}_{\mathcal{B}}(r) = \frac{3}{2} - \frac{7\ell^2}{2(r^2 + \ell^2)}, \tag{B3}$$

$$\bar{h}_{\mathcal{H}}(r) = 2 - \frac{9M\ell^2}{r^3 + 2M\ell^2}, \tag{B4}$$

$$\bar{h}_{\mathcal{C}}(r) = 1 - \frac{5\ell}{2(r+\ell)}. \tag{B5}$$

Based on these explicit expressions, the equation $\bar{h}_i(r) = 0$ admits real positive solutions for r, indicating a divergence of the effective metric tensor components $\bar{g}_{\theta\theta}$ and $\bar{g}_{\phi\phi}$. The existence of a curvature singularity may also be confirmed via examination of the Ricci (or Kretschmann) scalar corresponding to Eq. (B1), i.e.,

$$\begin{aligned}\bar{\mathcal{R}}_i(r) = {} & \frac{2\bar{h}_i\left(2\bar{h}_i^2 + 2r^2 f_i'\bar{h}_i' - r\bar{h}_i(4f_i' + rf_i'')\right)}{2r^2\bar{h}_i^2} \\ & + \frac{f_i\left(4r\bar{h}_i(3\bar{h}_i' + r\bar{h}_i'') - 4\bar{h}_i^2 - 7r^2(\bar{h}_i')^2\right)}{2r^2\bar{h}_i^2},\end{aligned} \tag{B6}$$

where primes denote differentiation with respect to r and explicit dependencies of the functions $f_i(r)$ and $\bar{h}_i(r)$ on r have been omitted for the sake of simplicity. Evaluating Eq. (B6) for each of the three models yields

where primes denote differentiation with respect to r and explicit dependencies of the functions $f_i(r)$ and $\bar{h}_i(r)$ on r have been omitted for the sake of simplicity. Evaluating Eq. (B6) for each of the three models yields

$$\bar{\mathcal{R}}_{\mathcal{B}}(r) = \frac{\mathcal{A}_{\mathcal{B}}(r)}{(r^2+\ell^2)^{7/2}(3r^3 - 4r\ell^2)^2}, \tag{B7}$$

$$\bar{\mathcal{R}}_{\mathcal{H}}(r) = \frac{\mathcal{A}_{\mathcal{H}}(r)}{2(r^3+2M\ell^2)^3(2r^4 - 5Mr\ell^2)^2}, \tag{B8}$$

$$\bar{\mathcal{R}}_{\mathcal{C}}(r) = \frac{\mathcal{A}_{\mathcal{C}}(r)}{2r^2(2r-3\ell)^2(r+\ell)^5}, \tag{B9}$$

which diverge at $r_{\mathcal{B}} = \frac{2\ell}{\sqrt{3}}$, $r_{\mathcal{H}} = \left(\frac{5}{2}\right)^{1/3} M^{1/3}\ell^{2/3}$, and $r_{\mathcal{C}} = \frac{3\ell}{2}$, respectively, and the numerators are given explicitly by

$$\mathcal{A}_{\mathcal{B}}(r) = (r^2+\ell^2)^{3/2}(9r^8 - 69r^6\ell^2 - 268r^4\ell^4 - 384r^2\ell^6 - 96\ell^8) + 2M(57r^8\ell^2 + 250r^6\ell^4 + 336r^4\ell^6 + 192r^2\ell^8), \tag{B10}$$

$$\begin{aligned}\mathcal{A}_{\mathcal{H}}(r) = {} & 16r^{15} + 8M(84M - 43r)r^{11}\ell^2 + M^2(3726M - 2879r)r^8\ell^4 + 2M^3(5040M - 4517r)r^5\ell^6 \\ & + 80M^4(60M - 137r)r^2\ell^8 - 2800M^5\ell^{10},\end{aligned} \tag{B11}$$

$$\mathcal{A}_{\mathcal{C}}(r) = \ell^2\left(-5(r+\ell)^3(7r^2 + 30r\ell + 18\ell^2) + 2Mr^2(71r^2 + 12r\ell + 216\ell^2)\right). \tag{B12}$$

Review Article

Stay in touch!

International Journal of Modern Physics D | Vol. 31, No. 09, 2230015 (2022) | Review Paper

Black holes and their horizons in semiclassical and modified theories of gravity

Robert B. Mann, Sebastian Murk and Daniel R. Terno

https://doi.org/10.1142/S0218271822300154

arXiv:2112.06515 [gr-qc]

Int. J. Mod. Phys. D **31**, 2230015 (2022)

Next >

PDF/EPUB

Tools Share Recommend To Library

Abstract

For distant observers, black holes are trapped spacetime domains bounded by apparent horizons. We review properties of the near-horizon geometry emphasizing the consequences of two common implicit assumptions of semiclassical physics. The first is a consequence of the cosmic censorship conjecture, namely, that curvature scalars are finite at apparent horizons. The second is that horizons form in finite asymptotic time (i.e. according to distant observers), a property implicitly assumed in conventional descriptions of black hole formation and evaporation. Taking these as the only requirements within the semiclassical framework, we find that in spherical symmetry only two classes of dynamic solutions are admissible, both describing evaporating black holes and expanding white holes. We review their properties and present the implications. The null energy condition is violated in the vicinity of the outer horizon and satisfied in the vicinity of the inner apparent/anti-trapping horizon. Apparent and anti-trapping horizons are timelike surfaces of intermediately singular behavior, which manifests itself in negative energy density firewalls. These and other properties are also present in axially symmetric solutions. Different generalizations of surface gravity to dynamic spacetimes are discordant and do not match the semiclassical results. We conclude by discussing signatures of these models and implications for the identification of observed ultra-compact objects.

Keywords: Semiclassical gravity ▪ modified gravity ▪ black holes ▪ apparent horizon ▪ evaporation ▪ white holes ▪ energy conditions ▪ thin shell collapse ▪ surface gravity ▪ information loss

Google Scholar

Inspire HEP

LinkedIn

ResearchGate